The Logic of Lacan's Not-All

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Abstract: This article offers an analysis of Lacan's notorious formulae of sexuation. Lacan develops these formulae on the basis of the classical Aristotelian square of opposition. Nonetheless, attempts to localize the formulae within such a square, lead to all sorts of logical inconsistencies. Therefore, the first conclusion of this text is that these formulae cannot be placed on the four, relationally-determined corners of a square. A second thesis starts with the simple observation that Lacan's formulae consist of two pairs of two equivalent propositions, as a result of which there is, at first sight, no difference between the two, so-called masculine and feminine sides of Lacan's diagram. The difference, however, does not concern a negation (e.g., p and -p), but a difference in use of the quantifiers. This goes for both the universal, $\forall$, and existential quantifier, $\exists$; in this article only the former is discussed. Derived from a critical examination of works by Peirce, Frege, and Blanché, Lacan introduces a distinction between the extensional and intensional interpretation of a universal quantifier. This distinction allows to interpret $\forall$ in '$\forall x \Phi x$' extensionally (Φx applies to all x) *or* intensionally (x is essentially Φ). This distinction implies that there may be no exception to $\forall x \Phi x$, but also that one cannot exclude that there is an x to which Φ does not apply as an identifying feature. Moreover, it divides the all into an all and a *not-all*, where the latter does not negate the former, but reveals the illusion that 'all' subjects are defined necessarily by the function Φ.

Keywords: Formulae of sexuation, Aristotle, square of opposition, Peirce, object *a*, extension, intension

I learnt very early on that logic is capable of incurring the world's odium.[1]

Introduction

Although Lacan's formulae of sexuation have often been discussed before, commentaries on the logic pertaining to them are rather rare.[2] The reason for this, unsurprisingly, has to do with Lacan's undidactic presentation of them. Simply put, there is no single *écrit* or part of the

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1 Lacan 2017, 86.

2 Inspirational were Copjec 1994, Fink 1995, Verhaeghe 1996, Žižek 1991, 1993 and 2012, and Zupančič 2017; for the logical aspect in particular, see Loparic 1991, Badiou 1992, Cathelineau 1998, Cejvan 2014, Grigg 2008, Le Gaufey 2006 and Chiesa 2016. I wish to thank Adrian Johnston and Dany Nobus for their helpful comments on an earlier draft of this text, and Lorenz Demey for his patient and illuminating answers to questions regarding the 'maximal' interpretation of particular propositions and its effect on the square of opposition (see below).

Seminar that summarizes in an accessible way what is at stake in them. For a discussion of the formulae, many readers have recourse to Seminar XX, but this seminar[3] is actually the last one in a series starting with the unpublished Seminar XV (1967-1968) – with, as we shall see, antecedents dating back to at least the early sixties –, during which the logical square is being constructed, detours included. Therefore, in order to articulate what is at stake in the formulae one needs to consider the seminars preceding Seminar XX. Which is not an easy task, for the formulae do not concern just one aspect of psychoanalytic theory – sexuality – but are related to most if not all of Lacan's fundamental ideas regarding psychoanalysis: its relation to science, its ethics, discourse theory, the question of desire, enjoyment and love, the difference between speech and writing, etcetera. Therefore, this article's aim cannot be but a modest one, that is to situate the formulae within the broader context of Lacan's work and to detail their *logic*.

No 'relation'?

The problem that the formulae tackle can be put in fairly simple terms: if there is sexual difference, and if this difference is of such a nature that it does not allow for a relation (*rapport*) between the two sexes, how to conceptualize this difference? In what way do the feminine and the masculine differ from each other, so to make absent any sexual relation between them? Given the everyday experience of love and desire between the two sexes, leading to all sorts of phenomena that can hardly be named other than *relational*, how to make sense of Lacan's famous *dictum* 'il n'y pas de rapport sexuel'?[4] Does such a statement amount to more than a presumably profound, pessimistic wisdom regarding the inevitable difficulties, not to mention the solitude, human beings encounter in the realm of sexuality? And more generally, how to assess such a statement within a theoretical framework within which and at its most fundamental level, 'relations' play a pivotal role?

A simple reminder of the canonical definition of the signifier as "what represents the subject for another signifier"[5] teaches us that the signifier does not exist on its own, but only in *relation* to another signifier. More precisely, as Lacan reminds his audience during the same lesson, a signifier is nothing but what differentiates it from other signifiers and therefore only 'is' what other signifiers are not.[6] There is only a signifier to the extent that it occupies a place within a relation to other signifiers; a relation, moreover, that precedes it.

Besides this, the so-called 'mathemes' Lacan provides his readership with, are not unlike mathematical formulae in using letters (instead of words open to diverging interpretations), but also share with them the quality of establishing *relations* between those letters.[7]

And, finally, the grid onto which the formulae of sexuation are inscribed, derives from Aristotle's logic, which, again, consists of *relations* between different formulae. This becomes clear if we take a look at the Aristotelian square of opposition, containing four formulas. In themselves they express already a relation between a subject (S) and a predicate (P) – differing qua *quantity* (all or some) and/or *quality* (affirmative or negative), allowing for four possible combinations, A (all +), E (all -), I (some +) and O (some -) – but they become logically even more relevant through the relations they entertain with one another. These relations facilitate simple derivations – from "All men are mortal" (A), one can conclude that "Some men are mortal" (I); if "Some men are mortal" is true, then "No men are mortal" (E) is definitely false, etcetera – but also more complex forms of syllogistic arguments.[8]
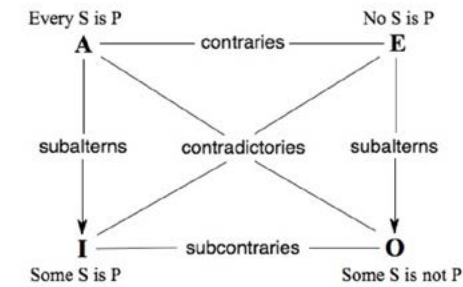

FIGURE 1: Aristotle's square of opposition[9]

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terms."

7 See Burgoyne 2003 and Corfield 2002 for a critical examination of these mathemes.

8 Kneale and Kneale 1962, 67ff.

9 Parsons 2017. It should be noted that one will not find this 'square of opposition' in Aristotle's works, the diagram was constructed by the 2nd century philosopher Apuleius, based on his reading of Aristotle's *Organon*. Although the square fell into disuse in the twentieth century – due to the introduction of the formal logic developed by Frege, Russell, and many others – it continued to be discussed in the writings of Jacoby 1950, Blanché 1957,1966 and others, eventually leading to a complete research programme on logical geometry; see http://logicalgeometry.org. Jean-Yves Béziau wrote an accessible and entertaining article on issues related to the square and its more recent transformation into a hexagon. Here, one paragraph is worth quoting at length: "The problem of the O-corner is quite different from the problem of the I-corner. The question is not a wrong-name problem but a no-name problem. This has been pointed out especially for the case of the quantificational square: it seems that there are no natural languages in which there is a primitive name for 'not all'. In such situation, linguists speak about 'non-lexicalization'. A radical view would be to argue that if there

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3 Lacan 1998 [1972-1973].

4 The first literal occurrence of this statement can be found in Lacan 2006b [1968-1969], 226, although there are precursors such as "Le sexe, dans son essence de différence radicale [...]", cf. Lacan 1964-1965, 19 May 1965.

5 Lacan 1961-1962, 6 December 1961.

6 Cf. Ferdinand de Saussure 1966, 120: "[I]n language there are only differences without positive

The relations between the formulae can be defined as follows: "Two propositions are *contradictory* iff [= if and only if] they cannot both be true and they cannot both be false. Two propositions are *contraries* iff they cannot both be true but can both be false. Two propositions are *subcontraries* iff they cannot both be false but can both be true. A proposition is a *subaltern* of another iff it must be true if its superaltern is true, and the superaltern must be false if the subaltern is false."[10] So, at first sight, the logical square of opposition seems to be a rather unfortunate choice for any possible explanation of the thesis according to which there is no sexual *relation*. This becomes even more problematic if one takes the appearance of Lacan's version of the square into account.
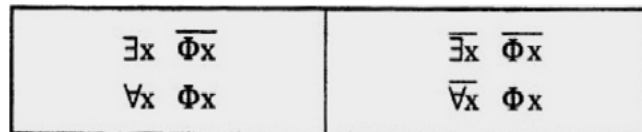
$$\exists x \ \overline{\Phi x} \qquad \overline{\exists x} \ \overline{\Phi x}$$
$$\forall x \ \Phi x \qquad \overline{\forall x} \ \Phi x$$

FIGURE 2: Lacan's formulae of sexuation[11]

One discerns two sides, a left and a right one, textually clearly identified, respectively, as the male and female side.[12] So, in addition to the relationality inherent in the square of opposition, Lacan's re-elaboration of it seems to involve the most classical of binarisms: the one and its other, separated into two distinct, juxtaposed halves.

This not only runs counter to what seems to be at stake in the formulae – a non-relational difference – but should also be situated against the background of a more general problematic. First of all, a constant theme within Lacan's work concerns the object-pole of the subject-object relation, and more in particular that this object should not be understood as the thing that would make one complete. Surely, the object *a* is often considered as a lost object, entailing the idea that it could be found again or somehow recuperated. Yet, the proper Lacanian

additional qualification of this lost object is that it is *structurally* lost and that the classical Freudian idea of a paternal castration threat forbidding access to the incestuous maternal object, is nothing but a prohibition of what is anyway *impossible* qua object of desire. In that respect, the Lacanian approach of desire is at odds with a long, dominant Western ontological, ethical and political tradition – from Aristophanes' myth about human beings as divided into two halves desperately searching one another in order to reconstitute a whole, up to a certain *das Wahre ist das Ganze* – which reserves an important place for what one may call 'plenitude' or 'wholeness'. The issue, however, is to conceptualize this structural incompleteness both against and within a tradition that advocates one or the other version of 'wholeness'. One can state and repeat that the object is structurally lost, proclaim the subject to be a divided subject etcetera, but the challenge remains to *argue* for this in a theoretically solid way.

With the formulae, the issue can be put as follows: how to conceptualize a difference that makes any universalization impossible, a difference as radical that it is not based upon a common ground or is played out against the background of a *unifying* domain? This problem seems to have preoccupied Lacan from the very beginning up and until the last phase of his work. In the imaginary, the basic operation is (mirroring) identification, through which all are equal. Sexual difference on this level, formalized as $-\varphi$, is about having or not having a penis and suggests a potential complementarity between those who have and the have-nots.[13] Within the symbolic, difference is the operational principle, yet this difference is ultimately guaranteed by one specific signifier, the phallus. As the signifier of lack – the negation at work within the symbolic system – the phallus both establishes the link with the signified or (sexual) meaning *and* bars any definite access to it. Put differently, in Lacan's structuralist account of the unconscious the Saussurean sign becomes stripped of its unity, relocating the signifier and the signified to two separate realms, kept apart by a bar. This function of the bar, i.e. to prevent the two realms from blending into one, is taken up by the phallus. Although this allows for a take on the subject as lack or as marked by the bar of castration – hence the notation $ for the barred subject –, within this frame of thought, sexual difference seems to be a mere symbolic issue of a division into two positions. Regarding the phallus different (symbolic) positions are possible[14], but that does not change the basic conception of the subject

are no names for something, it does not exist, or that the notion has no meaning." Béziau 2012, 7. The author is referring to the O-corner of the square and the fact that all (or most?) natural languages have words for A, E and I, but not for O. E.g. 'always' (A), 'never' (E) and 'sometimes' (I) form a useful triangle of three contrary propositions. Enlarging this triangle into a square of opposition implies the addition of a fourth proposition, O, for which adequate words seem to lack, which compels to the detour of adding a negation to the A-corner – in our example this would be 'not always'. Because of its connection with Lacan's not-all, in the closing paragraphs of this chapter we will return to this issue.

10 Parsons 2017, my italics.

11 As noted above, it took Lacan several years to construct this square and hence there are different, incompatible versions of it. Here, the one discussed is the final version, included in Lacan 2018 [1971-1972], 95, 118, 178, 183 and Lacan 1998 [1972-1973], 78.

12 Lacan 1998 [1972-1973], 79-80.

13 Women, according to Lacan, are not sexually desirable for a man because of their genitalia, but because of their motherly physical characteristics, breasts and hips; see Lacan 1998 [1972-1973], 7 and 35.

14 Cf. Lacan's earlier work, according to which masculinity and femininity are different *with respect to each other* (i.e. in a *relational* way), in the sense that the one and the other involve, respectively, having the phallus (on the basis of initially not having it and of possibly losing it) and being it.

qua subject of the phallic law. As a subject of the symbolic, the subject may be thoroughly sexual, but this sexuality is inscribed into the symbolic and is nothing but a series of possibilities to situate oneself with regard to the phallus, based on a primordial affirmation of a symbolic order organized by the phallic law.

In Lacan's later work phallic determination remains the basic idea when it comes to conceptualizing desire and enjoyment.[15] Figure 2, the table of the formulae of sexuation, includes one (and only one) function, the phallic function . Therefore, if one is looking for 'a jouissance beyond the phallus'[16], a supposedly exclusively feminine enjoyment, one wonders where to look for this in a (logical) universe made up of *one* function ( ) and *one* variable (x).

The real difference
Influenced by Alexandre Koyré[17] on this point, Lacan considers mathematics to be the defining characteristic of modern science's *modernity* and does not refrain from introducing it into his theoretical apparatus. However, it is only logic that provides a basis for mathematics, examining its method and its proofs, including inferences such as 'if… then…' and connectives such as 'and' and 'or'. It is a this point that one can situate a domain, shared by both psychoanalysis and modern science, namely logic. As Lacan put it early on: "[…] all what psychoanalysis is about, is of the order of language, that is, in the end, a logic."[18] At a later stage of his teaching, and most clearly within the context of the formulae, logic is considered as providing an access to the real, for "[…] it is only because of logic that there is an access to the real."[19] This may sound surprising, for wouldn't one expect the 'real' – certainly if one distinguishes it from 'reality' – to be something outside and different than the manipulation and connection of symbolic notations, such as p q? This

becomes more intelligible if one adds that Lacan equates the real with the impossible. A passage in Seminar XVII is unambiguously clear about this: "[…] the real is the impossible. Not in the name of a simple obstacle we hit our heads up against, but in the name of the logical obstacle of what, in the symbolic, declares itself to be impossible."[20] In brief, the real is the impossible and one encounters it as a 'logical obstacle' in the symbolic at its purest, that is, logic. As he puts it in Seminar XIX: "[…] the real I've been speaking about is accessed via the symbolic. We access this real in and through the impossible that is defined only by the symbolic."[21] This may provide some answer, but also shifts our question to: what is this 'logical obstacle', this impossible *real* within the symbolic, qualified as an impasse of formalization?[22]

'Obstacle' is not a novelty in Lacan's work. As Jacques Derrida's pupils point out in their *The Title of the Letter*[23] – a work praised and repeatedly referred to in *Encore* – the notion of 'instance' in Lacan's early text, 'The Instance of the Letter'[24], should be related to the Aristotelian notion of *enstasis*, meaning 'objection' or 'counter-argument'.[25] In 'The Instance of the Letter' (and elsewhere), this objection is clearly a phallic objection or obstacle, in the sense that the field of desire is characterized by a structural and constitutive lack, symbolically incarnated by the phallic signifier. The subject is not 'one' and fundamentally lacking, despite all the (psychoanalytic) myths about love as fusion or about, conversely, the birth trauma as the painful undoing of a unity with the maternal body. The phallus is, as noted above, the obstacle to any oneness, to any obliteration of difference (i.e. the endless metonymy of the signifier), or to any erasure of the bar separating signifier from signified. As Lacan reminds his public in *Encore*: "[…] the function of the bar is not unrelated to the phallus."[26] Despite its de-unifying function, the phallus is a bad candidate to incarnate the 'logical obstacle' Lacan is looking for. On the one hand,

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15 Lacan 1998 [1972-1973], 79-80.

16 Ibid., 74.

17 See: "[…] ce stupéfiant effort pour expliquer le réel par l'impossible – ou, ce qui revient au même – pour expliquer l'être réel par l'être mathématique parce que, comme je l'ai déjà dit, ces corps qui se meuvent en lignes droites dans un espace vide infini ne sont pas des corps *réels* se déplaçant dans un espace *réel*, mais des corps *mathématiques* se déplaçant dans un espace *mathématique*." Koyré 1966, 185-186 and 199. The 'impossible' Koyré refers to here, concerns counter-intuitive notions such as 'empty space' and 'inertia' (of moving objects): one does not experience these, yet they are needed in order to explain scientifically empirical phenomena (an apple falling from a tree, for instance).

18 Lacan 2005 [1953], 59. From an early stage onwards, Lacan is quite serious about logic; and 'logic' here means logic in its most technical and austere sense. Hence his lukewarm response to people referring to his logical investigations (and not only his topology) as 'an elastic logic' and his repeated criticism of fellow psychoanalysts who think they can do without logic, because Freud stated that the unconscious is *Widerspruchslos*, that it does not know negation or contradiction. Lacan 1961-1962: 21 February 1962, Lacan 1967-1968: 24 January 1968.

19 Lacan 1978, 23.

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20 Lacan 2007 [1969-1970], 123. Many other passages could be quoted in support of this identification of the real with the impossible, to which only logic provides access. E.g. "[…] the impossible that turns out to be the real, because it can only be founded within logic." Lacan 2001, 439; or "For a long time now I've been asserting fairly clearly, for it to be enough to recall it here, that the real […] is af-firmed in the impasses of logic." Lacan 2018 [1971-1972], 29. For a discussion of the place and import of (logical) formalization in Lacan, see Cutrofello 2002 and Livingston 2017.

21 Lacan 2018, 121-122.

22 Lacan 1998 [1972-1973], 93.

23 Nancy and Lacoue-Labarthe 1996 [1972], 24.

24 Lacan 2006a [1966], 412-441.

25 For more about the context and reception of Nancy and Lacoue-Labarthe's book, see Rabaté 2011; for *enstasis* in particular, see Chaitin 1996, 111, 144f, and Cejvan 2014.

26 Lacan 1998 [1972-1973], 39. In the context of a discussion of *enstasis*, Lacan refers to courtly love, which is a sublimation in the sense that it keeps the object of desire at a distance – it raises an 'ob-stacle' to any fulfillment of desire – and finds (phallic) enjoyment in encircling it. Ibid., 69.

because it is considered to be a function – which is not an obstacle, but on the contrary, a fundamental operator within a logical universe – and on the other hand, because the phallus may install lack and undo unity, but it does not seem to entail an exception to its universality. So, if there is an 'other jouissance', a feminine exception to the universality of the phallic function, and given Lacan's avoidance of simply stating that women are not subjected to the phallic function – which would run counter to his fundamental definition of *any* subject as a subject of the signifier – what sort of exception are we dealing with? The latter is definitely not a simple negation, but rather an obstacle.[27] And can we find this feminine jouissance beyond the phallus *qua* obstacle within logic, as Lacan seems to be convinced of?[28] Lacan's contention is that this obstacle is a symbolic obstacle, not in the sense of the phallus as a detotalizing function, but as a logical impasse. Lacan identifies two of these impasses, namely the 'not-all' and the 'at-least-one' (*hommoinsun*).[29] Before going into a discussion of these impasses, we first need to acquaint ourselves with the domain within which these impasses occur, i.e. logic.

Peirce's quadrant and Frege's judgement

As mentioned above, Lacan's formulae of sexuation are constructed in a dialogue with the Aristotelian logic and square of opposition. This logic was dominant from ancient Greek times until the end of the nineteenth century. Despite this dominance, most philosophers struggled with several issues pertaining to the square. The most notorious one concerns the so-called existential import of logical statements. If one states that 'all men are mortal' does this imply that 'men' exist? If so, how about 'all unicorns have one horn'? The former statement is obviously true – it is, what Kant would call, an analytical statement, in the sense that the notion of 'unicorn' necessarily implies 'one horn' – but one may be reluctant to conclude that unicorns exist. The example of the unicorns may lead one to conclude that statements of the form 'All S are P' have no existential commitment.[30] Yet, that does not solve the problem within the Aristotelian square, for as we have seen above, there is a relation of subalternation

27 "[...] it is not at all a matter of making one the negation of the other, but on the contrary of one standing as the *obstacle* to the other." Lacan 2018 [1971-1972], 86; my italics.

28 "[...] the jouissance that people call by whatever name they can find, the other one, precisely, the one that I am trying to get you to approach by a logical pathway, because, as things currently stand, there is no other [than a logical pathway]." Lacan 1998 [1972-1973], 75.

29 Cf. Lacan 2001: 479: "Recourir au *pastout*, à l'*hommoinsun*, soit aux impasses de la logique." Throughout this article *pastout* is rendered as *pastout* and not as *pas-tout* or as *pas tous*. This is in accordance with the former quote and stresses the unity of the negation and the quantifier 'tout'. Surely, Lacan uses variations such as *pas toute*, but his first aim is to introduce a new quantifier, *pastout*, and not so much to state that 'not all [two words] x...' or that x is 'not-whole'.

30 Which is certainly not Aristotle's position on this issue, as his logic is clearly an onto-logic, i.e. statements of the form 'all S' imply the existence of at least one S.

between the A- and I-corners. This means that if "All S are P" is true, then "Some S is P" is also true. The latter formula – the I-corner – explicitly suggests that there is an S (e.g. 'unicorn') if one holds 'Some S is P' to be true. In brief, it seems that Aristotle's logic forces one to hold certain statements to be true, although one might not want to do so.

The problem of existential import was eventually solved elegantly by Charles Sanders Peirce. In his chapter of *Elements of Logic* dealing with the square, Peirce proposes the following division of the four basic propositions, A, E, I and O.[31]
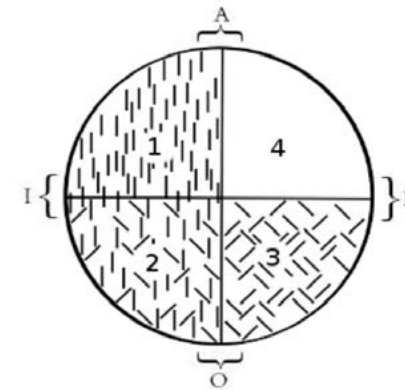

FIGURE 3: Peirce's diagram

In the example Peirce uses, the subject is 'line' and the predicate is 'vertical'. Remarkable here, is that the empty quadrant 4 of the diagram, is shared by both the A and E propositions. This means that universal statements, either positive (A) or negative (E), do not necessarily entail the existence of the subject term (in this case, 'line'). They are both trivially true if their subject does not exist. Only propositions of the I and O form imply the existence of their subject term.

Peirce's diagram may solve the issue of existential commitment, yet the price one pays for it is that the Aristotelian relations between the propositions disappear. Whereas Aristotle considered A and E to be contrary (cannot both be true, but can both be false), Peirce names them "incongruous or disparate"[32], for they are different, yet they can both be true (cf. quadrant 4) and both be false (cf. quadrant 2). Subalternation is no longer a valid inference (from A to I, or from E to O); I and O can no longer be called 'subcontraries'. The only surviving relation is the one of contradiction (both cannot be true, yet one must be) between, respectively, A and E, and O and I.

31 Peirce 1933 [1893].

32 Ibid., 459.

For our purposes it is worth noting that Lacan uses this diagram throughout his Seminar, starting from the ninth, that is *L'identification* (1961-1962).[33] In those sessions the repeated take on Peirce's diagram consists in pointing out how the affirmative universal (A) is valid both when all S are P (quadrant 1) and when there is no S at all (quadrant 4). This leads to a reformulation of A into 'there is no line which is not vertical'. This double negation makes explicit the operation required to actually arrive at a subject: either there is none, or one needs two determinations (or negations), namely two predicates, 'line' and 'vertical'. The subject, 'line', only becomes identifiable if one adds a second predicate, 'vertical'. Were one to isolate the pure subject (line), stripped of its verticality, then the subject is lost, and one shifts to quadrant 4: if there is no verticality, there is also no line. Interestingly, here the 'impossible' occurs for the first time: if the subject belonging to quadrant 1 were to ask 'is there a line without being vertical?', the answer it gets from quadrant 4 is 'not possible'.[34] In brief, the imagined exception (the 'not possible') of quadrant 4 not only confirms the rule of quadrant 1, but *creates* the rule of quadrant 1: the general identification of the subject 'line' with the predicate 'vertical' is only possible by *excluding* the possibility of the *absence* of the predicate 'vertical' (quadrant 4).[35]

This 'no line which is not vertical' is true when there are either only vertical lines, or no lines at all. This basic structure of the A proposition – which defines the subject, for it says what it is and how one can recognize it – is approached in a different way by Gottlob Frege. Although Lacan's emphases are similar to the ones regarding Peirce's diagram, from the brief overview below we will gain one additional and crucial element that makes the formulae of sexuation readable.

The introduction of Frege's logic in the fifteenth Seminar, *L'acte psychanalytique* occurs rather abruptly but is also no surprise.[36] In the preceding lessons, logic already appeared, in particular the distinction between the classical, Aristotelian logic and the modern one, initiated by Frege, amongst others.[37] The ambition of Frege's *Begriffschrift* consists in purifying logic of any trace of natural language or what he calls the *Sprache des Lebens*.[38] This attempt at stripping language of any ambiguous meaning effect or 'content', in order to solely preoccupy oneself with its form, is not a novel one. Already in Aristotle's work – the *Analytica Priora* – the initial impetus to formalization is present, yet the application of (meaningless) letters remains rooted in the Greek language, with all sorts of odd and obscuring effects. Frege's intention, however, does not consist in formalizing language to such an extreme that one would be left with a totally 'empty' formal language. He rather wants to develop a new language, apt for pure thought. In this new language concepts *(Begriffe)* are expressed, but only as a writing of logically well-formed sentences and of their interrelations. In this respect, his logic resembles mathematics, but as a *method* it is situated at the more fundamental level of elementary notions such as 'and', 'if', 'not' and so on. In general, the aim is to arrive at a *Lückenlosigkeit der Schlusskette*, that is to create a language that allows for a watertight connection between logical sentences.[39] In order to do so, only two connectives are required, negation (*Verneinung*) and implication (*Bedingtheit*, the conditionality of "if ... then ..." clauses), added to the general form of assertions:

FIGURE 4: the basic form of an assertion[40]

33 In this seminar Lacan discusses Peirce's diagram in the lessons of 17 January, 7 March and 27 June 1962 (mentioning Peirce's name not even once!). Other occasions include Lacan 1967-1968, 7 February, 28 February and 6 March 1968 and Lacan 2006c [1971], 69.

34 Lacan 1961-1961, 7 March 1962. See also: "Little Hans, who is as much of a logician as Aristotle, postulates the equation *All animate beings have a phallus*. I assume I'm addressing people who followed my commentary on the analysis of Little Hans, and who also remember what I took care to accentuate last year [that is in the Seminar on Identification, 1961-1962] concerning the proposition known as the universal affirmative, namely, that the universal affirmative is only meaningful in defining the real on the basis of the impossible. It's impossible for an animate being not to have a phallus." Lacan 2014 [1962-1963], 78.

35 On a formal level this argument is analogous to Jacques-Alain Miller's reading of Frege's theory of number: one only arrives at one (1), at something countable, by excluding an impossible concept *and* including it as an empty set, that is zero (0). See Miller 1966 and also Lacan's notion of alienation: subjectivity is based upon a forced choice for one (vertical) out of two options, vertical or non-vertical. If it were to choose non-verticality, there would also be no subject. In that sense, the neurotic subject is a reply from the real (Lacan 2001, 459), for its symbolic subjectivity is based upon an excluded impossibility (the real), yet this exclusion needs to be reckoned with as *cause* if one wants to understand *neurotic* subjectivity, i.e. a subjectivity not fully coinciding with its symbolic identifications (or logical predicates); see also Lacan 1961-1962, 21 March 1962.

36 Lacan 1967-1968, 13 March 1968.

37 Frege's theory of number was already alluded to in Seminars IV and IX; see Lacan 1994 [1956-1957], 237 and Lacan 1961-1962, 28 February 1962.

38 Frege 2014, xi.

39 Ibid., x. Lacan reminds his audience of the contemporaneity of Freud's psychoanalysis and Frege's foundation of modern logic; cf. Lacan 2018 [1971-1972], 41. It is tempting here to connect Frege's desire for a language without gaps (*lückenlos*) to Freud's development of psychoanalysis, based on a taking into account of the gaps (*Lücken*) in the continuity of conscious mental activity. See, e.g., 'Manuskript K' (1896), which deals with hysteria and its primary symptom, i.e. the *Schreckäusserung bei Psychischer Lücke*; Freud 1986, 177.

40 Ibid., 24.

An assertion consists of a horizontal line (the *Inhaltsstrich*, content stroke), preceded by a small vertical one (which indicates that the sentence is an *Urteil*, a judgement, meaning that one holds the assertion to be true). At the end of the sentence Frege places the function which applies to the argument (or variable) for which a concavity (*Höhlung*) is reserved in the middle of the horizontal line. This concavity is the place for what one traditionally called the subject. However, Frege's reformulation of the subject-predicate logic into a logic involving functions and arguments has the advantage that no existential commitment whatsoever is made. Sticking to our initial example of 'all lines are vertical', 'vertical' and 'line' become, respectively, function and argument. If there are no lines, the concavity remains empty, which does not change the proposition's truth. Moreover, and more clearly than is the case with Peirce's diagram, there is no essential relation between argument and function. The function 'vertical' can be applied to any x that is vertical. In that respect, Frege's logic is a purely *extensional* one: the proposition is true for any object that may happen to be vertical. A function does not tell us anything about the 'essential' characteristics of the argument ('subject'), it simply creates a set (an extension) consisting of all elements that satisfy the function.

It seems that Lacan makes the effort to introduce his audience to Frege's logic – not without reassuring them that this is tied up closely with the psychoanalytic clinic – to illustrate his definition of the subject as that what a signifier (S_1) represents for another signifier (S_2). In Frege's rearticulation of Aristotle's logic the bond between argument (S_1) and function (S_2) is loosened to the point that together they may form a valid logical proposition, but there is no longer any essential bond expressed between them. 'Line' (S_1) may be the argument of the function 'vertical' (S_2), but nothing prevents any other S_1 to take up the place of the argument. This is well made visible by the concavity in Frege's assertions: it is a void waiting to be filled by one or the other argument.

Despite this resemblance between Frege's propositions and Lacan's take on the subject, it is precisely when pointing this out, that Lacan focuses on a difficulty, already present in Aristotle, which Frege does not touch upon, let alone solve. His logic may be purely extensional, the question is how something comes to occupy the place of the argument (= the concavity in the assertion). For this to happen one always already needs 'something' one is talking about, or the existence of what logicians call 'a domain'. Propositions like 'All lines are vertical' imply that potentially there is an x which can be identified as 'line', and more generally, that it is 'lines' we are talking about. This may raise the question what a line is, to which the answer can only be put in a logical proposition of the form "All lines are …", suggesting an infinite regress – any proposition seems to require another one. That is why Lacan wonders whether the subject (= argument) does not appear as a first predicate

(= function), since the x of a logical proposition already seems to imply a well-defined collection of such x's.[41] Within the context of the formulae of sexuation, this fundamental question is repeated when Lacan is commenting on the formula $\forall x \; x$: "What is this *x*? I have said that it is defined as though by a domain. Even so, does this mean we know what this is? Do we know what a man is when we say *all men are mortal*? We learn something about him from the fact of saying that he is mortal, and precisely from the knowledge that this is true for all men. However, before introducing this *all men*, we only know the most approximate features, which can be defined in the most variable fashion."[42]

This observation, of course, does not impair Frege's logic, but touches upon a more general issue which, as we will see, proves to be crucial for Lacan's construction of the formulae of sexuation.

On why the square is not a square
Before presenting our reading of the formulae, we need to discuss one more logical aspect. Apart from Frege and Peirce, Lacan[43] also pays tribute to Jacques Brunschwig, who published a technical article on Aristotle's logic in the last issue of *Cahiers pour l'analyse* (1969), a journal founded by Jacques-Alain Miller, amongst others.[44] In this article, Brunschwig points out an ambiguity in Aristotle's use of 'some'. 'Some' may mean 'some, perhaps all' – this is its so-called minimal interpretation, allowing for subalternation between A and I – or rather 'some, but not all'. The latter, 'maximal' interpretation corresponds to the ordinary usage of the word 'some', but logically it has quite a dramatic effect on the square of opposition. If one opts for the maximal interpretation of 'some' – 'some, but not all' – then subalternation between the universal and the particular is no longer possible. Instead of being able to derive the particular from the universal, A and I entertain a relation of contrariness (both can be false, but not true together). The relation between the two particulars, I and O, changes as well, from subcontrariness into equivalence, for if, e.g., one states that 'some birds are black' (I) – and here, 'some' means 'some, but not all' – then one implies that 'some birds are *not* black' (O), and *vice versa*.

There seem to be good reasons to relate Lacan's *pastout* – i.e. one of the 'logical impasses' referred to above – to this maximal interpretation

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41 Lacan 1967-1968, 13 March 1968; see also Lacan 1961-1962, 17 January 1962.

42 Lacan 2018 [1971-1972]: 32-33 and 6.

43 Ibid., 90; see also Lacan 2016 [1975-1976], 6, and 184-185 for Jacques-Alain Miller's clarifying note, including more references.

44 For a summary of Brunschwig's article, alongside translations and comments on many of the other articles included in *Cahiers pour l'analyse*, see the resourceful website http://cahiers.kingston.ac.uk put together by Peter Hallward, Knox Peden *et al*.

of 'some'. First of all, it forms an obstacle or objection to the universal proposition including the same subject and predicate; and what we are looking for is, as noted above, an *obstacle* to universality. Secondly, in most contexts the first meaning of this some is 'not-all'; if one states that 'some flowers are red' one may first and foremost intend to state that '*not all* flowers are red'. So, isn't Lacan's (feminine) *pastout* a direct application of Brunschwig's insightful distinction between, on one hand, the *minimal* 'some', privileged by Aristotle, allowing for the traditional square of opposition, and, on the other hand, a *maximal* 'some' objecting to universal propositions and saying 'no' to 'all', which sounds exactly like the de-universalizing proposition one may want to attribute to the feminine op-position to the universality of the phallic function? This is Guy Le Gaufey's guiding thesis in his detailed analysis of the formulae, which will be discussed in the following paragraphs.

Le Gaufey's logical approach to the formulae eventually results in this square of opposition:
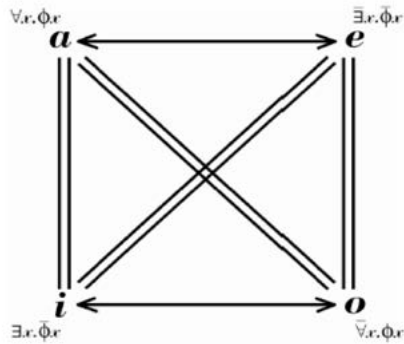

FIGURE 5: Le Gaufey's version of Lacan's square[45]

The reader immediately notices the differences with Lacan's presentation of the formulae (see Figure 2). First of all, the formulae are identified according to the Aristotelian distinction between A, I, E and O; secondly, relations are added between the formulae, namely = (contradiction) and ↔ (equivalence), and thirdly, the formula $\forall x\, \Phi x$ has switched places from the bottom right to the top right of the square. The advantage is obvious, for it provides – as the title of Le Gaufey's book promises – a logical consistency to Lacan's formulae. Although all three alterations deserve a critical examination – and, in my opinion, should be rejected – as they are based on Le Gaufey's initial decision to read Lacan's formulae as a square of opposition in which the particulars, I and O, are interpreted in a maximal way, the discussion will be limited to this.

45 Le Gaufey 2006, 87.

For his decision, Le Gaufey finds inspiration in the aforementioned article by Brunschwig, who constructs the following diagram:
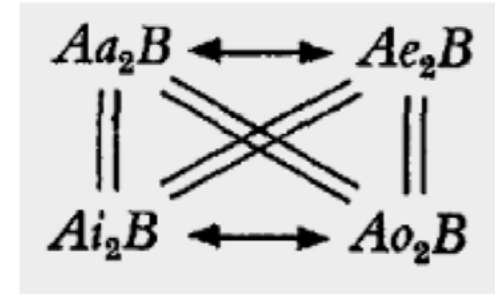

FIGURE 6: square of opposition with maximal I and O[46]

Despite the differing symbolic notation – e.g. 'Aa_2B' instead of "All A are B" – this square is indeed identical to the one including Lacan's formulae, as Le Gaufey presents it. That we are dealing with a maximal particular is made clear by the subscript '2' included in each formula, which is intended to differentiate it from the more common minimal interpretation and use of the square. The point not to overlook, however, is that Brunschwig presents this diagram in a conditional mode: "*if* one wants the couples a-o and e-i to remain contradictory, [...]"[47] As we have seen, in the Aristotelean square the universals A and E are contradictory with, respectively, the particulars I and O. If we interpret the latter in a maximal way, then these – I_2 and O_2 – are not only contradictory to their diagonally opposed universals – E_2 and A_2 – but also to the universals on the same side. This logically results in the equivalence of A_2 and E_2. At first sight, there are some problems with this reasoning. If we start at the bottom of the diagram, the equivalence of I_2 and O_2 is logically sound: if one defines 'some' as 'some, but not all', then 'some A are B' is equivalent with 'some A are not B', as both propositions mutually imply each other. If we now move to the relations between, respectively, A_2 and I_2, and E_2 and O_2, then it is less clear why one would call them contradictory. Contradictory means that both propositions can be neither true, nor false together – simply put, it is either the one or the other. Yet, an example makes us doubt this idea immediately: if I_2 is 'some trees are blue', then A_2 is 'all trees are blue', which makes it not too hard to imagine that *both* propositions can be false, i.e. when 'no trees are blue' (E_2). If both propositions can be false, then they are not contradictory but contrary. And if A_2 and I_2 are contrary, and if – as we have seen – I_2 and E_2 are contradictory, then A_2 and E_2 cannot be equivalent, for they entertain

46 Brunschwig 1969, 7.

47 Ibid.

different relations (contrariness and contradiction) to the equivalent I₂ and O₂. In brief, Brunschwig's diagram looks like a forced and an altogether false construction of a 'maximal' square. Yet, this is too hasty a conclusion, for Brunschwig was careful enough to start the presentation of his 'maximal' square with a conditional *if* and to conclude it by pointing out the *paradoxical* nature of this square. Moreover, in the subsequent paragraph and in a clarifying footnote, makes clear that A₂ and E₂ are *both* defined as 'all are or all are not', which simply means that A₂ and E₂ are not so much equivalent, but rather identical. The same holds for the particulars I₂ and O₂, for the maximal 'some' means in both cases 'some are and some are not'. This leads Brunschwig to conclude that, given the identity of A₂ and E₂, and of I₂ and O₂ the maximal square is not a square of opposition but "a segment of opposition" involving two instead of four propositions.[48]

When extension and intension do not coincide
Brunschwig's remark about the 'maximal' square of opposition serves as a good starting point for this concluding section on Lacan's formulae of sexuation. For, indeed, many commentators mention the fact that from a classical logical point of view Lacan's four formulae can easily be reduced to two.[49]
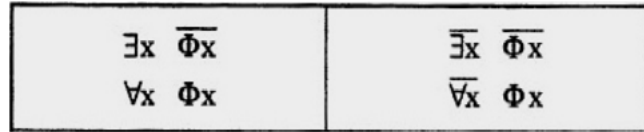

FIGURE 2: Lacan's formulae of sexuation

In this diagram, $\forall x \ \Phi x$ and are equivalent, as are Φx and $\exists x$. Stating that 'all x are subjected to Φ' ($\forall x \ \Phi x$) means exactly the same thing as stating that 'there is no x which is not subjected to Φ' ( ), and the same conclusion applies to Φx and $\exists x$.[50] So, why would one complexify things, using two pairs of equivalent formulae contradicting each other, instead of two contradictory ones? Or, put differently, why state *one* thing *twice*, i.e. $\forall x \ \Phi x$ *and* , Φx *and* $\exists x$?

The first thing to observe here, is that if Lacan is logically articulating a difference it is definitely not a binary opposition; there is no opposition whatsoever between the 'two' sides, as both express (logically) one and the same opposition between a formula containing an existential (or particular) quantifier, $\exists$, and a formula containing a universal one, $\forall$. If there is any difference at stake, it can only be found in the interpretation of the quantifiers, in particular the $\forall$.[51]

On the left side, we find an unambiguous assertion according to which 'all x are subjected to Φ' ($\forall x \ \Phi x$). This proposition is contradicted by the one above it, stating that 'there is an x that is not subjected to Φ' ($\exists x$). As already mentioned before, here we can locate the *real* of the symbolic universe whose subjects are all subjected to the phallic function. This universe, as Lacan argues, is only possible on the basis of the exclusion of the *impossible*, namely that there would be one who is not subject to the phallic law. The more technical point here, concerns the issue we encountered during our discussion of Frege, namely the constitution of a domain of x's. How do we know what belongs to this domain, or, how do we single out x's who serve as arguments satisfying function Φ? In many commentaries of the left, so-called 'masculine' side of Lacan's diagram, the x is identified as 'the male (subject)'.[52] Even if one adds that this x could just as well be a biological woman – for sex is not to be reduced to natural characteristics – this seems to be missing the point of Lacan's *logical formalization* of subjectivity and sexuality. The formulae on the left side express nothing more (or less) than that subjectivity – what one is dealing with in the psychoanalytic clinic – is defined by its being subjected to the phallic function. In order to constitute this domain of subjects, one needs to define it and here the impossible exception plays its role of providing the essential predicate, phallic castration, to be able to single out x's belonging to the domain. In pseudo-mathematical language one could state that the exception draws the circle around the elements that belong to the class defined by $\forall x \ \Phi x$. As Lacan puts it in one of the earlier seminars, any class presupposes a 'classing' (*classement*) and this does not happen through including elements to a pre-constituted class, but first of all by creating a class based on considering the absence of a certain trait as impossible.[53] This, moreover, makes evident the proximity and difference between

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48 Ibid., 8.

49 Yet, to my knowledge, none of them considers this awkward fact to be the key to 'unlock' the formulae. The apparent equivalence between the formulae is not a difficult yet secondary characteristic of Lacan's formulae (that one can ignore or should explain away), their equivalence is precisely the point of Lacan's logical argument.

50 Lacan puts the negation stroke above the quantifiers $\forall$ and $\exists$ and the function Φ. As we will see, this is done for a specific reason.

....
51 That is another reason not to follow Le Gaufey's presentation of the square: a logical square is based on the difference between two sides, whereas Lacan's 'square' may be not a square at all, but rather a one-sided 'universe' within which two formulas are repeated, *with a small difference*. This difference is indicated by the vertical line separating the two pairs of formulas, equivalent along a diagonal axis. The vertical line, therefore, should not be considered as a clear-cut division into two, but rather as the virtual line separating inside from outside on a Moebius strip.

52 See, e.g., Fink 1995, 104-125.

53 Lacan 1961-1962, 7 March 1962.

logic and psychoanalysis. Both deal with the subject (the argument 'x' in assertions); and what this subject is, its essence, as Lacan puts it, "situates itself essentially in logic. It is a pure statement [*énoncé*] of discourse."[54] The difference, however, is that logicians usually take the domain (or the universe) to which their propositions apply for granted, and if, as Lacan argues, the constitution of a domain requires an impossible proposition, then this impossibility, this 'impasse' is – for sound reasons – left out of the discussion. The *neurotic* subject is in that sense not a logical subject, for it entertains a complex relation – Oedipal, Freudians would argue – to this excluded impossibility or the *hommoinsun*.[55]

Moving to the right side, we have to address the difficulty that its two formulae are seemingly equivalent to the ones discussed above. This impression, however, is immediately contradicted by the divergent translations of one of Lacan's crucial notions pertaining to the right, 'feminine' side of the diagram. This notion is *pastout*, and one can find at least three different translations in English of it: not-whole, not-all and notany.[56] As the latter is a mere suggestion, the more interesting alternatives are not-whole and not-all, especially because they appear in officially translated volumes of Lacan's Seminar. The first option, not-whole, is the one chosen by Bruce Fink in his translation of *Encore*; the second one, of more recent date, is Adrian Price's choice, in his translation of *... or Worse*, for 'not-all' as the English equivalent for *pastout*. In a translator's footnote, Price qualifies Fink's argument in support of his 'not-whole' "as flimsy as it is unprecedented".[57] There is indeed no immediate reason to render Lacan's *pastout* into 'not-whole', for it seems to be a mere, although oddly written, negation of the classical, Aristotelean 'all'. Yet, without going into the details of Fink's argument, it is also clear that Lacan does not conceive of his *pastout* as a negation of 'all', i.e. a 'not every'. If one reads *pastout* as 'not every', one can only interpret Φx as 'not every x is subjected to the phallic function', which is equivalent with the aforementioned ∃x and turns Φx into a mere redundancy. Therefore, one can argue in favour of Fink's choice to avoid the quantitative 'not-all' (or 'not-every'), although the major disadvantage of 'not-whole' is its tempting suggestion to interpret the proposition as 'x

is not-wholly subjected to the phallic function', leading to (or inspired by) the ubiquitous commentaries on Lacan's feminine as a subject that is not wholly castrated or, as a subject including a part that subtracts itself from castration.[58]

The obvious retort to this sort of reading is that one does not need logic to state, let alone argue, that men and women, with respect to the phallic function, are different. This may be a clinical observation, but this also amounts to considering sexual difference as a fact. While, as we have seen, sexual difference may be empirical or clinically observed, its 'reality' can *only* be approached via logic. In brief, if we follow Lacan's torturous ways through the complexities of classical and modern logic, one should provide *logical* arguments for the basic intuition that any sexuality is hetero-sexuality, that is dividing the asexual unity of the domain, to which *all* subjects belong, into two.

The logical argument for the *real* of sexual difference is indeed related to Lacan's introduction of the *pastout*, one of two 'impasses' mentioned above. Again, it is fruitful not to limit one's reading to *Encore*, but to also pay attention to the first more or less sustained discussion of *pas tout*. This occurs in the last session of Seminar IX, *L'identification*.[59] There, Lacan makes use of the well-known saying 'all that glitters is not gold' (*tout ce qui brille n'est pas or*). This saying is a rather odd one, for one should not take its formulation literally to mean that all that glitters is not gold, but rather as *not all* that glitters is gold.[60] Lacan mentions that he is not the first to discuss the phrase, but keeps his audience ignorant about who else may have shed some light on this negation of a universal statement. There is no conclusive evidence for this, but one can guess that Lacan is referring to Robert Blanché's article on negation and opposition, for in that text it is argued that there are two ways to the negate the affirmative universal, namely to deny either universally, or universality.[61] The author remarks that it is difficult to render this difference into French and therefore expresses it in Latin, *omnis non* and *non omnis*. He explains this in the following way: "We have recourse to Latin in order to avoid the equivoques of French, which usually does not put a negation in front of a complete proposition [that is *non omnis*]. Thus, *Tout ce qui brille n'est pas d'or* appears as a negative universal [...] although the meaning is obviously

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54 Lacan 2006c [1971], 109. Here we touch upon a topic also present in Lacan's later seminars, that is the difference between the said (*dit*) and the saying (*dire*). One can relate this to the issue of universality and its constitutive exception, in the sense that what is said (the universal) relies on a primordial saying (the exception) that provides a symbolic being (an essence) to any x (or the subject) belonging to the domain.

55 Lacan 2001, 479.

56 See Lacan 1998 [1972-1973], Lacan 2018 [1971-1972] and Gallagher 2008, respectively.

57 Lacan 2016 [1975-1976], 229. It should be noted that Fink's translation of *pastout* as 'not-whole' runs into trouble immediately after its first occurrence in *Encore*, obliging the translator to an expression of surprise: "Curiously enough, he [Lacan] says *pas-tout* here instead of *pas-toute*." Lacan 1998 [1972-1973], 7. See above, footnote 29.

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58 Despite Lacan's statement that: "She [a woman] is there [in the phallic function] in full (*à plein*)." Lacan 1998 [1972-1973], 74.

59 Lacan 1961-1962, 27 June 1962; anticipated in the lesson of 17 January 1962.

60 One could relate this to the O-corner's difficulty discussed in footnote 9: it would be easier to state that 'some glittering things are not gold', but if one wants to emphasize that it is definitely *not* all of them, there is no immediate word available, hence the addition of a negation in front a proposition that starts with 'all' (and which may explain this negation's capacity to move from its proper place, in front, to a less logically precise position in the sentence).

61 Blanché 1957, 190-191.

a particular one, negating universality. It is a pity [...], if one wants to avoid any possible equivoque, to be forced to compensate for this via detours such as *il n'est pas vrai que...* (or, in English: *it is not case that...*)"[62] So, if the problem revolves around the absence for a word that expresses directly 'it is not the case that all ...' – cf. the O-corner discussed above – Lacan's *pastout* can be considered as filling this gap in the French language, albeit with a specific twist.

In his comments on the French saying, *Tout ce qui brille ...*, Lacan indicates that it would be missing its point to simply understand 'gold' as a subset of 'glittering', implying that all what is gold glitters, but not the other way around. It is rather that there is something about gold that makes it glitter – "what gives to an object its desirable and fascinating colour" – yet, the saying makes us doubt not just anything that glitters, but gold itself. If gold is the cause of our desire for anything glittering, if 'glitter' only interests us to the extent that one supposes it to be gold, then if not all that glitters is gold, this puts into question gold. In that case, one does not doubt the existence of gold, but wonders what recognizable (i.e. desirable) feature it has beyond its glittering quality. Simply put, if not all that glitters is gold, what is gold then? Is there a way to tell false from true gold, if only 'glittering' is available as a discriminating feature? And if gold is the cause of glitter, the reason why we are fascinated by glitter in the first place, should we actually trust this cause, knowing that as cause of (our desire for) glitter it is able to produce glitter which is not gold? As Lacan puts it with a pun, the saying hints at the not-gold (*le point d'orage*) of gold.

This saying and the questions it elicits on Lacan's behalf, should be related to the paragraphs right before them, one of many passages in which Lacan discusses the Aristotelian 'all' and the question whether it establishes between subject and predicate a relation of *having* or rather *being*. This difference is known in logic as the difference between *extension* and *intension*, or, as Lacan prefers to name them, *étendue* and *compréhension*.[63] In most cases Lacan seems to reject the distinction or, at least to doubt whether it is of any use to consider a proposition as referring to objects that satisfy the function (*extension*; they *possess* the required predicate) and/or as expressing a meaning (*intension*; they *are* what the proposition expresses about them).[64] This reluctance to embrace

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62 Ibid., 190-191.

63 He quotes this couple of notions from the Jansenist treatise on logic, *La logique ou l'art de penser*, published in the year of Pascal's death; Arnauld and Nicole 1992 [1662], 51-52. See also Kneale and Kneale 1962, 318 and Blanché 1970, 184. Note: Lacan connects *compréhension* to the 'time for comprehending' introduced his 1946 text on 'logical time'; Lacan 1964-1965, 13 January 1965. In an e-mail exchange (30 Sept. 2018), Dany Nobus pointed out that both *compréhension* and comprehending involve a question about *being*.

64 Lacan 1961-1962, 17 January 1962; Lacan 2018 [1971-1972], 101.

the distinction between extension and intension, should not surprise us, for 'comprehension' (i.e. intension) is precisely what Lacan considers secondary to and often irrelevant for his approach of the subject as a subject of signifiers (barred from their signified). If one states, e.g., that all human beings are mortal, "what does one understand then? [...] What is there to understand?", Lacan asks rhetorically. However, this does not mean that Lacan will simply leave this distinction aside. On the contrary, it is one of the themes most constantly returned to throughout the construction of the formulae. The crucial issue here, concerns the problem we already discussed before: how does a domain – or a 'universe of discourse' as Lacan sometimes calls it as well, borrowing this terminology from the logicians Augustus de Morgan and George Boole – gets constituted, in order for objects to be(come) elements of it, and, conversely, how does one single out objects in order for a domain to get constituted? Regarding the so-called 'male' side, we have argued that this is done by a constitutive exception, allowing for the universality of 'All x is subjected to the phallic function'. This means that the domain consists of x's that satisfy the function Φ. Therefore, in this case, the *extension* and *intension* (the '*compréhension*' or 'meaning' referred to above) of $\forall x\, \Phi x$ mutually imply each other. Its extension is all the x's belonging to the domain, its intension concerns the definition of x, what this x *is*, or in less logical terms, what allows us to recognize such an x as an x belonging to the domain – which is, in this case, 'being subjected to the phallic function'. There is no other trait, characteristic, attribute or predicate (*intension*) that allows us to count an x as an x belonging to the domain (*extension*). This is, of course, in conformity with the basic axiom of Lacanian psychoanalysis: a subject is a subject of castration, and there is no other definition available or any other predicate that would give an x 'access' to the domain.

This distinction was already known to Aristotle, for he differentiates between three different meanings of the universal 'all': 1. *kata pantos*, 2. *kath'auto* and 3. *katholou*.[65] The first use is identical to what is currently named extension; the second one refers to intension – the predicate expresses something essential about, a 'defining' aspect of the subject – and the third use expresses what one calls a 'commensurate universal', relating a predicate to all subjects (*extension*), in an essential way (*intension*), establishing co-extensivity between the subject and its predicate.[66] The latter criterion simply means what we have been describing above regarding the Lacanian subject: all subjects are castrated and all what is castrated is a subject. One could also describe it as the case where subject and predicate are indistinguishable, not

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65 Aydede 1998, 24.

66 Ibid., 28.

even allowing for questions about the possible difference between the extension and intension of $\forall x \, \Phi x$.

This, however, is precisely what the 'feminine' side of Lacan's diagram does: seemingly, it is a mere repetition of the 'masculine' side, yet it drives a wedge between the extension and the intension of $\forall x \, \Phi x$. This reading is corroborated by several passages in which Lacan refers to the difference between *pan(tès)* and *holon*, or, in Latin, between *omnis* and *totus*, or, in modern logical language, between *extension* and *intension*.[67] The *pastout* read in a classical way, may lead one to conclude that it is equivalent with $\exists x$.[68] Yet, as we have seen, the *pastout* is not a straightforward denial of affirmative universality – it definitely does not mean $\exists x$ or $\forall x$ – it means that 'all x are subjected to the phallic function' *extensionally* but not in any *intensional* way.[69] The quantifier *pastout*[70] – invented by Lacan in response to Blanché and other logicians dealing with the problem of ex- and intensionality, and rendered as a negation not in front, but above the $\forall \, (\Phi x)$, *pastout* in one word and not *pas tout* or *pas tous* – indicates that all x are subjected to the phallic function (*extension*), but do not constitute an 'all', that is a class or a domain defined by this particular function (*intension*). All the elements of the *set* defined by Φx do not constitute a *class*. In a way analogous to the example discussed above, 'not all that glitters is gold', the *pastout* puts the function (gold or, in this case, phallus) into question: if the phallus determines subjectivity – turns *all* x into a subject – then the *not-all* introduces doubt whether the phallus suffices to do so. This does not mean that one can use any other function than the phallic one to identify the Lacanian subject or, conversely, that there are subjects who are not subjects of the phallic function. The mere difference between $\forall x \, \Phi x$ and Φx resides therein, that the latter proposition allows for subjects of the phallic function who are not *essentially* characterized as such. If one were to identify those as 'women', one can state that they do have an *existence*, yet no *essence* as subjects of the phallic function. The latter idea is expressed in many different ways by Lacan, from the famous *La femme n'existe pas* – there is no *the* woman, for this would imply the possession of an essential trait[71] – to more oblique formulations such as for women the phallus does not function as *cause* (i.e., it does not determine them as subjects), they are

not linked to castration essentially[72] or the *pastout* is "an all outside of 'universe'" (*un tout d'hors univers*), that is not the negation of 'all', but an 'all' that negates its own capacity to constitute a universe (or domain).[73]

Conclusion

Whether there is a non-binary difference between men and women, was the guiding question in our step-by-step reconstruction of the formulae. Now, we can locate this difference in the *pastout*, which is "an objection to the universal".[74] This universal, Φ, concerns both men and women as subject; they may relate differently to it, but that does not alter their status as subjects of the phallic function. The difference, therefore, is not a mere negation of the function Φ, for, on the contrary, this function has as its argument *all* x. The difference or 'logical impasse' concerns the constitution of the domain to which *all* x belong. On the 'masculine' side of the formulae, there is a short-circuit between what the subject is and what it has qua discriminating feature, i.e. phallic castration. This side is not contradicted by the other, 'feminine' one, but simply opened up by a formula that says 'yes, all are subjected the phallic function, but no, that is not what we *are*." Or, put differently, men's existence coincides with their essence (= symbolic being), whereas women, as the other sex, suggest the possibility of an existence without an essence.

How is this related then to the absence of sexual relation? Paradoxically enough, sexuality is the field where the partner appears as the *a*-sexual object of desire. A sexual relation would imply that it is based upon or produces a common ground between two different positions. Yet, as we have seen there is no difference, only an obstacle dissolving any (shared) universality. Here we encounter the two dimensions of the object *a*. On the one hand, it is the ir-rational object, the part without a whole, the thing that exists without having a being (i.e. a symbolic essence), and as such the incarnation of the obstacle to universality of the symbolic. On the other hand, it is a phantasmatic object, creating the illusion of a possible completeness. Sexuality may be *not all*, the object *a* occupies the place of and veils the *notall* in the phantasm.[75]

67 Lacan 1961-1962, 17 January and 27 June 1962; Lacan 1967-1968, 13 March 1968.

68 Lacan 1998 [1972-1973], 102: "In other words, this not-all, in classical logic, seems to imply the existence of the One that constitutes (*fait*) an exception." (translation modified)

69 Ibid. 103: "[...] it is no longer from the perspective of *extension* that we must take up the not-whole." (my italics)

70 Ibid., 72: "[...] a never before seen function in which the negation is placed on the quantifier".

71 Ibid.

72 Lacan 2018 [1971-1972], 35.

73 Lacan 2001, 466. This is 'confirmed' by the other 'feminine' formula, , which is equivalent with $\forall x \, \Phi x$, but also denies that there is a constitutive exception to the universality of $\forall x \, \Phi x$. All women are subjected to the phallic function, but there is no universe (*intensionally* speaking) of 'woman'.

74 Lacan 1998 [1972-1973], 102.

75 Lacan 1967-1968, 28 February and 20 March 1968. This allows for an interpretation of Lacan's seemingly offhand allusion to Pascal's well-known *pensée*: "[...] anyone trying to act the angel acts the beast." Lacan 1998 [1972-1973], 20; Pascal 1966, 242; see also 60. In sexuality the other appears as an *a*, with whom one performs an 'angelic', sexless act, only to (re-)produce the mute (*bête*) signifier that governs the subject as a subject of the signifier. This universal coincidence of angelism and animality, however, is supplemented by a contingent *not-all*.

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