

“To be and not to be – that is the answer”: Paraconsistency and Dialetheism According to G. Priest

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Abstract: Ever since Kurt Gödel put forward his incompleteness theorem, an important number of non-classical forms of logic have emerged. Some of these, such as paraconsistent logic, have tried to come up with novel ways of thinking the relationship between logical consistency and completeness. Among these, some, especially the one developed by Graham Priest, have been developed with the explicit goal of vindicating the Hegelian dialectics, and showing its relevance at the formal level. In this text, I will begin by discussing the historical and philosophical debates that existed prior to the emergence of paraconsistent logic, by making extensive allusions to Albert Lautman’s reflections on mathematical philosophy and the conceptual or metaphysical residue of all mathematical formalization. I will then propose a critical appraisal of Priest’s work on paraconsistency, of his *Dialetheism*, by especially focusing on whether it is possible to formalize dialectics, i.e. to successfully capture it at a mere formal level. Discussing this question will necessitate examining, in the concluding section, the uncertain and complex relationship between science and especially formalization on the one hand, and politics and negativity on the other, where I will analyze the formalization of dialectics alongside and in relation to the institutionalization of communism.

Keywords: Paraconsistent Logic, Dialetheism, Formalization of Dialectics, Negativity, Politics and Science, Graham Priest, Albert Lautman

“To be *and* not to be – that is the answer”¹
G. Priest, 2006

This study follows up, in relation to the example of paraconsistent logic, an older one published in 2010 entitled “Lautman’s Duality Against the Hegelian Negativity, and the Paradox of Their Formalizations”. Classically, a contradiction in the logical sense of the term is the conjunction of a formula and its negation (or logical contradiction). A formal system is called “consistent” when we cannot deduce such a conjunction on its basis, otherwise it is called “inconsistent”. The principle of paraconsistence consists of subverting this absolute exigency² of consistency, all by avoiding inconsistency: this is done by

1
G. Priest, *Doubt Truth To Be A Liar*, p. 208.

2
Founded at the level of Being by Aristote 2008 I, 3, 1005b, 15-30.

allowing, logically, a certain kind of contradictions. Different versions of paraconsistent logic exist, and the latter is merely one of the many ways of doing “non-classical”³ logics. The common trait of all these different forms of non-classical logics is their calling into question, on a variety of different points, the dominant paradigm since Aristotle until Frege and Russell. What is special about paraconsistent logic is that many of its versions have been produced with the explicit goal of justifying Hegel, and it is in the name of an active self-defense technique of dialectics that I am interested in it. This means that this study, which deals with a singularity within logic, is as incomplete as biased⁴, and it takes on a very exploratory dimension.

The introductive section 0 will state that all formalizations of dialectics, as far as they propose a particular interpretation of negation and of contradiction, intervene at the heart of the very idea of logic. Section 1 will posit, first of all, the problem of formalization in its general form, as an undertaking producing retroactively the criterion of demarcating a “speculative” conceptuality from a “positive” conceptuality, from, in other words, a conceptuality potentially rationalizable within the canons of deductive exactitude: what is at stake is to show, essentially, that the very idea of formalization, from the point of view of that which it tries to formalize and the goals that it follows by doing so, gathers in itself all the problems of the moving frontier between the philosophical territories and the (logic)-mathematical ones, and that this problem is above all, and always, a historical one. I will treat, secondly, the formalization of dialectics specifically, by trying to show how in this case the formalization business comes up against antagonistic suspicions which reveal the explosive character of its paradoxes. Section 2 will therefore present the paraconsistent logic in its general lines, the way it has been theorized, syntactically and semantically speaking, by G. Priest, and will dwell, in particular, on examples taken from “dialetheia”, in other words, dialectical contradictions which are real and/or logically acceptable, and which bring him, immediately, to intervene *stricto sensu* at the ontological level, by articulating a metaphysical monism and a praxeological conception of Truth. Section 3, rather short, will take stock of the study, and will compare the obtained results with those drawn in 2010 concerning the works of Doz-Dubarle. I will try, briefly, to analyze the fact that, essentially, these two destinies of the negative’s

3 Cf. the panorama provided in Priest 2001.

4 For a vision of all paraconsistent theories, cf. Norman, Priest & Routley 1989a; Batens, Mortensen, Priest & Van Bendegem 2003.

formalization make it suffer in the same way – they dissolve it -, even if the two road’s difference reveals two very different visions of the problem and of its stakes. Finally, in section 4 I will attempt to present a history of the problem in its different strata, that I will then extend, in a tentative fashion, by a politicization, both brief and radical, of the whole affair.

When Marcuse - partly in the wake of Lukacs for whom mathematics was the most advanced objectified form of reification, both of them relying on Hegelian maxims on the rigidity of the thoughts of Understanding -, undertook his radical critique of the “positive” one-dimensional philosophies, and defended the “bi-dimensionality” of dialectical logic against formal logic, this is because the latter was, in his eyes, the prototype, since Aristotle, of logics of domination and of submission to the established order. There is, he used to say already in Reason and Revolution, under Hegel's authority, “an intrinsic link between mathematical logic and unconditional submission to facts”⁵. We subscribe to this idea; however, in Reason and Revolution as in the One-dimensional man, Marcuse, like most of the dialecticians who work along the same lines, does not live up to his ambitions (and who displays, in the end, the same shortcomings as those who combat dialectics by spreading grotesque prejudices about it). All defenders of the negative thinking should force themselves to look closely into that which they want to deconstruct. And this is the reason why Lukacs or Marcuse are not Hegel and Marx: the latter two did try to enter, in detail, into the logico-mathematical question⁶, but the former two did not. It is in the spirit of the latter two that here we wish to contribute, on the occasion of this very particular question of the formalizations of dietetics, to the reflection of a Marxist point of view regarding the respective territories of science and philosophy.

0. Consequence, Negation, Contradiction at the Heart of “Logic”

First of all, some remarks about certain stakes both general and centered on the idea of logic, and about some constraints with which a “dialectical” logic is necessarily confronted are necessary.

Logic is traditionally presented as the theory of valid inference. When Aristotle defines syllogism as "... a deduction in a discourse in which, certain things being supposed, something different from the

5 Marcuse 1968, p. 190.

6 I especially tried to demonstrate that in my PhD in Barot 2004.

things supposed results of necessity because these things are so" (Prior Analytics), he testifies that **at the heart of logic we find the idea of consequence**. A distinction is then imposed: that Y be the logical consequence of X can be a necessary fact independently of our will. But it does not automatically follow from that point that our affirmations concerning the fact that Y is the consequence of X are sealed by evidence. Classical logic comes from Aristotle, and in *Metaphysics*, Γ, he establishes an unequivocal link between the real and the discourse, so that the latter be presented as an indisputable reasoning: it is not possible to affirm one thing and that which contradicts it from the same point of view and at the same time, because it is impossible for the real to have a property and to not have it from the same point of view and at the same time: the logical principle of non-contradiction relies on the ontological affirmation of the real's non-contradiction. However, classical logic is maybe not suitable for all forms of inference, that it authorizes certain inferences that we informally refuse as doubtful, or that it forbids certain inferences that we recognize as legitimate.

Let us consider briefly "the paradoxes of the material implication": that which characterizes classical logic is the fact that it determines the logical consequence, the implication, above all in term of preservation of the truth. If A implies B, it is because the truth of A implies the truth of B, therefore that it is not possible that A be true and B false. Whence is concluded the affirmation according to which A implies B if and only if we do not simultaneously have A true and B false. From then on, as soon as A is contradictory (false), A false implies B independently of what B is. That is how this "Ex falso sequitur quodlibet" can be illustrated:

"If today's Bonaparte is communist, then the earth is flat"
is therefore a logically true affirmation.

Classical logic considers this statement to be valid. Whence, since a long while, the existence of deviant, non standard, logics, which challenge, for example "Ex falso sequitur quodlibet", in different manners: one of them consists of saying that this principle relies on a simplistic vision of the falsity of A, in other words, on a simplistic vision of what a contradiction is, because given the fact that the system is trivialized by contradiction, anything can be deduced. We can thus distinguish between contradictions that render the system trivial, and those that do not do so, i.e. enrich the concept of contradiction. Another way, more traditional, consists of saying that in "Ex falso sequitur quodlibet", there is no link between the premises and the conclusion. At the heart of this plurality, there is the problem of the concept of consequence, of which we can

say that it is not unequivocal: we can think that we do not have in our possession, at the intuitive and pre-theoretical level, an unequivocal and determined concept of what a valid logical inference is. A vague concept, to put it simply, that is to say an absence of concept (as Frege would say it): whence, first important point, the necessity of formalization, and at the same time the possibility of a plurality of formalizations.

We can distinguish, first of all, between logical implication and material conditional: the second one, defined by its truth table, is often used as the first one's basis, for example in Quine for whom "implication is the validity of the conditional", for whom, in other words, logical implication is entirely based on the truth functions, the quantifiers and the variables. But we can approach the problem of consequence from another point of view: when we wish to demonstrate that Y cannot be the consequence of X, that it is impossible, this means that it is necessary that that does not be the case. This amounts to demonstrating that it is necessary that the negation of Y be the consequence of X.

The question of negation is at the heart of logic, as the particular and emblematic form of the problem of consequence. We saw that it was not unreasonable to think that it could be that there exists, in itself, no unique logical consequence that a formalization would capture and codify technically in a legitimately exclusive manner. Correlatively, we could therefore say now that there does not exist, necessarily, only one "negation" whose properties would be fix in themselves. "Negation" would then be a kind of Idea-enigma in the Lautmanian sense, a kind of undetermined: in Lautman, moreover, the relation of contrariety between pairs of notions composing dialectical ideas (continuous-discontinuous, local-global, structure-existence, finite-infinite, etc.) is relatively undetermined⁷; it is, in other words, characterized merely by a relation of polarity and of inversion, of opposition or of tension in the broad sense, in short, a relation of duality, the one, for example, between the Same and the Other; Being and non-Being (Nothing) of which the logical relation of contradiction would be nothing but one possible determination, and in any case derivative.

The whole problem is then to characterize, if it exists, the central rational kernel of negation. The formalizations of dialectics have all the common feature of making the standard, logical, notion of negation, which encloses many things, more complex: bivalence, that is to say the sharing out of legitimate statements among true (T) and false (F), the idea that F is the logical negation, i.e. in contradiction to T, and that there

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7 Cf. E. Barot 2009a, ch. II.

is an incompatibility between a statement and the one that contradicts it. Bivalence would be represented, first of all, by the excluded middle, and then by the principle of non-contradiction, which itself would be based on a principle of identity which is unavoidable. This is because things are what they are, and they are not what they are not, therefore that all affirmations with regard to them either correspond to them, or do not, and that therefore all affirmations corresponding to what they are, are incompatible with the contradictory affirmation.

Hence we see that behind consequence and negation, what is at stake is the relation of contradiction. We could believe that a dialectical logic would be justifiable by indicating that these central notions, before passing through the filter of a given theoretical framework, are relatively undetermined, something that would harm classical logics as legitimately as non-classical ones, in short that it would be the “ontological pluralism” of negation which would let us justify a dialectical logic. In reality, the opposite is true in Graham Priest:

“How does negation then behave? There is an easy way to settle this question. There is no such thing as negation; there are many different negations ... I do not think that this is a good response ... The theoretical object should be adjusted to the real object, and the way the latter behaves is not a matter of choice”⁸

We can, if necessary, imagine that such a realism can be adapted to a technical pluralism, and besides, this technical pluralism is a fact. But Priest stands by the idea that one should try to capture the relation of contradiction which unites two statements, and that the idea that one statement is the logical negation of the other can be founded. From the notion of contradiction, he easily deduces the excluded middle, the principle of non-contradiction, and defines classically the falsity of a statement by the truth of its negation. The problem emerges when we consider impossible situations, in which a statement can be simultaneously true and false, but especially effective situations, in our world, in which statements are simultaneously T and F. How can we reconcile the idea that two contradictory statements are simultaneously true with the idea that they are contradictory? Such is the problem, Hegelian par excellence.

For now, the important consequence is the following: if by the formalization of dialectics we mean the institution of a formalized logical dialectics, then the latter should technically clarify the concept

8 Priest 1999, took up again in Priest 2006a chap. IV “Contradiction”. I am translating.

of logical-dialectical consequence that it will call on, given that it will clarify the concept of negation, and will take a stand on the concept of relation of contradiction. And yet, given that these concepts of negation and contradiction are at the heart of the idea of logic, it follows naturally the affirmation that “formalizing dialectics”, whatever the precise sense that we attribute to dialectics be, is a way of taking a stand on the fundamental kernel of the idea of logic, this is because there is a native tension between dialectics and the dominant scientific regime of logos. We owe, naturally, our awareness of the meaning of this taking side to Hegel. But before continuing the discussion about the question of the formalization of dialectics, it is necessary that we analyze, first of all, the other side of the problem, which is not specifically logic, but rather transversally logico-mathematical: the problem of formalization in general.

1. Can We Formalize a Concept?

We cannot put forward the question “can we formalize a concept?” under the seal of the eternal: this very general question necessitates a detailed treatment divided between the philosophical and the mathematical fields, something that I will not undertake here. I am only going to try, by taking up again the Hilbertian lesson of the conference of 1900, to formulate as clearly as possible the problem contained by this question.

Asking whether we can formalize a concept is about (1) posing the problem of the respective identities of philosophy and mathematics, (2) posing the problem of the nature of mathematical objectivity (more than that of its “reality”), (3) identifying and measuring the historicity and the specific materiality of these two problems. It is only by relying on such bases that we will be able to show, afterwards, the organic character of these questionings by means of the limit example of the formalizations of dialectics⁹. The problem of the respective identities of philosophy and mathematics, to begin with, will allow me to approach, subsequently, the question of dialectics. And I will depart from the most immediate: both constituents of the general question, the “conceptual” and the “formal”.

9 By “mathematics” or “logic”, I mean to designate, following Tarski, the logico-mathematical in the broad sense: all that is related to a theory of the classes of objects and the relations that they can have, and that includes, at least, all classical propositional and predicative logics and set theory. This imprecision is naturally subject to caution.

1.1. The Conceptual and the Formal

1.1.1. Hazy Boundaries, Difficult Introjection

Even in a form of knowledge that is highly technical, there exist conceptual determinations, theses, in a more or less residual or implicit state, which deal with problems which cannot be transformed into “objects” or “methods”. It seems that it is impossible, by definition, to capture these problematic determinations in an unequivocal manner, because their equivocality, their ambiguity, their “enigmatic” character, is precisely that which makes them recalcitrant towards all such forms of closure. However, it is this equivocality which is at the origin of the famous theoretical inconsistencies that give rise, in their turn, to the willingness to rationalize in order to conquer once more the lost consistency: behind big crises, such as those that stemmed out of the well known tensions of the naïve concept of sets, understood as an indirect tool for rationalizing the number, those are the enigmas of the continuum and the uncountably infinite set which operate. This willingness to “rationalize” found an emblematic formula in the Hilbert program of the early 20th century, which consisted of self-rationalizing mathematics by means of finite formalization of all that was still of a speculative order. The “formalism” is here an attempt at turning the concept radically technical, with the aim of securing the edifice and making it exact: the initiative is about reducing the conceptual to the formal.

1.1.2. “Formalization”: General Definition

A preliminary definition is necessary: by the formalization of a given conceptual configuration I mean to designate, in a purely descriptive sense, this kind of *a posteriori* rationalization. We can define the initiative as a retranslation of the theoretical operators (“objects-oriented” or “methods-processes-oriented”) which compose it into a series of technically distinct and unequivocal operations, which are differentially identifiable at the syntactic and semantic levels, and of which the axiomatic form is the canonical legalization. The initiative resembles, in a generic manner, that of an *introjection* of concepts which are *de facto* para-formal, ante-formal, infra-formal or meta-formal, etc., the level that I call here “speculative” or “conceptual”, in a formal system, at the level of the “formal” or again of the “positive”. As such, the formalizing will relies, therefore, on two presuppositions: (1) these two levels are presumably *commensurable*, and (2) this commensurability is posited from the angle of the *reducibility* (total or sufficient) of the first to the

second, in other words, of the possibility of abolishing the initially “transcendent”, exterior, or “meta” character of the speculative.

Gödel's 1931 theorem¹⁰ forbids the foundational pursuit of this Hilbertian program of absolute reduction: one of the senses of the Gödelian incompleteness is that we do have “conceptual” forms which are irreducible to the formal, and which belong to mathematics. With Hilbert, the question of this reducibility was posed from the mathematical or meta-mathematical point of view. And yet, Lautman, looking at the impossibility of comprehensively formalizing the latter, considered this meta-mathematics to be in fact metaphysics. For him, the *speculative* as such emerged as the irreducible of mathematics itself, which is why he thought that the link between mathematics and metaphysics is not “contingent”, but necessary¹¹. Thinking this necessity is for him the task of “mathematical philosophy”. In other words, such formalization is not necessarily possible on the one hand and even when it is, it is not merely a “technical” operation on the other.

1. Being unequivocal and therefore exact can only be reached by starting, there also, from theses which discriminate, in the middle of the equivocality that is being dealt with, elements which are pertinent or meaningful, and by formalizing only the latter: here the technical work always responds to a question that logically precedes it.
2. Yet nothing can decide, in advance, if we are dealing with something speculatively irreducible as the result of which all attempts at formalizing it would, a priori, fail. Affirming the irreducibility or the reducibility of the conceptual to the formal cannot be made in advance. Which is why only the formalization effort can allow us to settle the question, according to its failure or its success. We cannot say, in advance, whether something can or cannot, in the middle of something enigmatic, give rise to a positive mathematical knowledge.

It seems therefore that there are speculative or philosophical elements in mathematics only insofar as there are, reciprocally, mathematical elements in philosophy. But is this a good way of describing the situation? There is, therefore, an important presupposition behind the question “can we formalize a concept?”: the distinction philosophy/

10 Cf. *infra*, section 2.

11 Lautman 2006, p. 237.

mathematics, which is precisely what we are concerned with. And the correlate of this presupposition (according to which the respective identities of mathematics and philosophy are clearly constituted), is then the one that concerns the nature of the “formalization” initiative: we cannot, from then on, reduce this question to one that is simply about connecting two distinct and exterior orders, more or less motionless, with rather stable contours, and which would allow to situate the question in advance and to characterize its stakes.

1.1.3 Displacing the Hiatus

The speculative, in short, is neither the mathematician’s prerogative, nor the philosopher’s, it is rather their common fate. A detour by Plato will be instructive. According to him, the difference between philosophy, in this particular case dialectics, and mathematics is related neither to their object (the intelligible, the universal, and the necessary, as opposed to the contingent, the particular and the spatial-temporal sensible) nor to their goal: exposing the object according to its concept, based on its internal necessity, its proper and “natural” law of development¹². Their difference is related to the fact that mathematics goes by hypotheses (circle, square, etc.), which means that it is marked by a finitude, the ignorance that any hypothesis, as position of existence, envelops. The dialectician refuses¹³ this way of stopping at the level of hypotheses, and insists on going beyond them in order to get to the anhypothetical (that beyond which we cannot regress anymore: the One Good). The dialectician demands that the object be exposed in its absolute necessity, which means the exhibition of its natural cause: the Idea, the suprasensible principle of being and of knowledge.

This is how we get to the categories of dialectics in the Platonist (but also Hegelian) sense - which emerge from Logos both as a discursive activity and as the essence of that which is (and this is how “Logic” rises up, equally in Hegel, in its ontological dimension, and even as ontology) -, which by definition are not simply at the crossroad of the two, philosophy and mathematics, or objects of an equal concern to both: infinity, one and multiple, totality, duality, etc., in short the intelligible is common to both mathematics and dialectics in that it precedes their

12 Cf. Plato 1997, VI, 509d-511e, the famous passage on the subdivisions of line, and the short dialogue Plato 1997, pp. 708-745 *Euthydemus*.

13 This is why dialectics is defined as the “science of free men” (*Sophist*, 253c): free in the sense of free from the opinion and the prejudices caused by the customary language, and here especially free from all constraints other than those of logos itself. The dialectician is never submitted to any law other than the one that thinking itself constructs in its movement towards its object (the essences and itself).

distinction. In other words, the problem of the difference between the speculative and the formal moves inside the speculative itself. And is there, this time, a way of differentiating between that which would be the philosophical conceptual and the strictly mathematical conceptual? The question is not purely rhetorical: it is not absurd to ask whether there are not “official” enigmas of mathematics, and enigmas which would be mathematical only in a secondary way or even not at all. If this is the case, we could consider the principle of formalizing speculative, “enigmatic”, proto-mathematical problems to be more legitimate than formalizing notions or categories which are not mathematical or not necessarily mathematical (for example, strictly “ontological” or even theological). In which case we could think that from the philosophical as well as the mathematical point of view, all willingness to formalize is not equally legitimate, beneficial or useful, that all formalization efforts do not have the same worth. But the only reason capable of establishing this kind of discrimination would then be the existence of a difference in the status or the origin of the incriminated “problem”, which would render “natural” the first one, but artificial, even useless or illegitimate, the second one. And yet, that implies that we establish an intra-speculative criterion of distinction: there again, that would presuppose that we would have already divided the territories between mathematics and philosophy. We notice that the difficulty which was initially linked to the conceptual-formal opposition has moved within the “conceptual” or the speculative.

Conclusion: this division, in the middle of the conceptual, between that which is mathematical and that which is not seems to be difficult, even impossible, to make, in so far as we do not have their distinction. And Lautman affirms this very same point in the period between the two wars. Following Plato, he renews and extends the dichotomy between the intelligible and the sensible, Being and beings. But he does introduce, compared to Plato, a number of important displacements. In particular, he establishes a functional homology between mathematics and the sensible, which leads to an ontological difference between the speculative enigmas, which he calls the Ideas (dialectical), and the Theories (mathematical). Consequently, metaphysics deals, for him, with these ontological “Ideas-enigmas”, which are supra-historical, and whose recurrent presence in history bears witness to their transcendence; as for mathematics, it produces theories which, in the historical context, are different sketches of solutions to these enigmas. Infinity, continuum, space, etc. are such enigmas, which have been present in history since Antiquity in the form of opposing couples (those already mentioned, finite-infinite, continuum-discontinuous, local-global, etc.) which are by themselves neither philosophical nor mathematical,

which are, in other words, both at the same time if we continue to talk on the basis of their distinction, but which are situated, in reality, at a logical level which precedes this distinction. When Bachelard says that the continuum is not an object but a concept¹⁴, or when Feferman says that the continuum hypothesis (CH) –or X’s move to $P(X)$ – is an intrinsically vague statement¹⁵, are they not saying the same thing? But we can take another example: imagine this other key concept, the concept of existence (with all the problems that it raises in constructivist terms, etc.). For Lautman, even if he is keen on structuralism à la Hilbert, or on the basis of its developments in the German algebra, or his friends and founder congeners of Bourbaki, the couple structure-existence (or essence-existence) is one such Idea-enigma, and not a couple of specifically mathematical notions¹⁶. If we finally consider the idea of “proof”, that is to say the discursive operation destined to justify an enigma by means of resolving a problem (rather than responding to a question), is it not equally prior to this distinction, because it refers to the generic exercise of the discursive rationality? Can we affirm that the idea of proof or demonstration is statutorily mathematical? The codified figures of sequential deduction (constructive, transcendental, etc.) constitute its adequate and exclusive formalizations? That is open to discussion.

We see here, in any event, that dialectics identified to meta-mathematics (and reciprocally) is the place par excellence of reflexivity. For Socrates-Plato, this methodological reflexivity remained par excellence the prerogative of dialectics, in so far as it goes beyond all hypotheses. But with Lautman, we see that if dialectics is really the only discourse capable of apprehending, of characterizing, this hiatus between itself and mathematics, in other words between the conceptual and the formal, it is for this reason that it is also a speculative intra-mathematical reasoning, in this particular case precisely meta-mathematical, as the protagonists of the “crisis in the foundations of mathematics” have testified to it.

In any case, as discourse of the connivance and of the mutual irreducibility of the speculative and the formal, this reflexivity (dialectical or meta-mathematical) appears as always necessarily situated,

14 Cf. Bachelard 1927, p. 221 and suiv.

15 Feferman 2000, p. 405.

16 Lautman illustrates this, in particular, by the relationships between non-contradiction (“Leibnizian “compossibility” of the “essences”) and existence, the adequacy between “structural” (syntactic) and “extensive” (semantic, i.e. in terms of domains of objects) points of view being non problematical in the finite case, notoriously problematical in the infinite case (in addition to the theorem of incompleteness from 1931, cf. the non-categoricity theorem of Lowenheim-Skolem).

historically contextualized and materially textualized. This intrinsically historical and practical sense of the initial question is entirely essential, and the constitutive historicity of the problem of the relationship between the speculative and the formal is precisely one way of making its treatment progress.

But for now, another aspect is going to allow us to advance, in the form of an acknowledgment: a very same speculative “conceptuality” touching these enigmas can give rise to different formalizations, whose compatibility is, moreover, not always immediate. We can logically see this possibility as a mark of an indetermination related to the enigma, indetermination itself expressing the fact that we are short of the distinction between philosophy and mathematics. And here we find, once again, the principle of the technical pluralism of negation and therefore of contradiction already mentioned in section 0.

1.1.4 Indetermination of the Conceptual and Over-(under) determination by the Formal

This indetermination is the fact that the relationships between contrary notions, in Lautman, are open to multiple realizations, to multiple moves to the formal, notably because the relation of “contrariety” is itself relatively undetermined: it resembles the *stricto sensu* logical or proto-logical relation of contradiction, for example in the couple finite-infinite, whereas this contrariety, in other examples, is clearly non logical: thus in the case of the couple structure-existence, or again local-global. For Lautman, the formal intra-mathematical diversity of theories, methods and domains of objects is the response that “the” mathematics “in its present development”, practical and historical, gives to this logically prior indetermination of the “Ideas-enigmas”. This is why he aims, at the same time, at explicating the way the unity of the enigmas is pluralized in a technical diversity, and at going back, regressing, on the basis of acknowledging this diversity, to the unity of the metaphysical questioning to which it responds. This double movement, descending from the one to the multiple, and ascending from the multiple to the one, it is the double movement of the Platonist dialectics, with which he identifies the mathematical philosophy, which is therefore not a philosophy “of” mathematics which would be applied, from outside, to an initially independent object: dialectics, in Lautman, but also in Plato and Hegel, is not a formal method, but is at one with its object¹⁷. Three

17 This is why, based on the object of inquiry (the one and the multiple in *Parmenides* whose introduction is provided by a discussion between young Socrates and Parmenides, on the occasion of the aporias related to the “ontological difference” between Ideas and sensible things – difference

remarks are now necessary:

(i) From that point of view we can notice that history goes strangely back and forth, even if this happens with regard to historically renewed modes and objects. At the “functional” level, there are recurrent “schemes” (“patterns” according to P. Kitcher¹⁸), structures of organization, modes of self-organization and of deployment, which are repeated in the historical movement of the production of theories. These schemes are like the structural conditions of historicity whose effectivity takes the shape of a compliance of mathematics itself with its own requirements of coherence: Noël Mouloud calls this the “teleonomy” of the evolutive knowledge¹⁹.

(ii) But at the level of “content”, there exists the same persistence of certain questionings through this historicity and its structures. This persistence is easily revealed through the evolution of the criteria that are used to demarcate that which is mathematical from that which is not, that is to say that which is recognized and legitimized as mathematical and that which is not. For example, the question of constructivity as a criterion for legitimizing a proof is emblematic of this historicity, but what is revealed here is the existence of a functional principle of discrimination, even if the content of this discrimination changes. In other words, what is raised by the process of formalization is the problem of mathematical objectivity as a principally historical problem: the becoming of its legitimization, its continued legitimization. It is important to mention that with regard to mathematical “reality”, Lautman insists on the necessity of avoiding all forms of “realism” but also all forms of “nominalism” of first kind: a mathematical reality is verified by its *facts* (a discovery, a theorem) concerning certain *beings* (or objects: functions, numbers, etc.), within the framework of theories determining and resolving certain Ideas-enigmas: all of reality is situated where these four “instances” or points of view meet, and this meeting point is necessarily dynamic, in other words there is a historicity of mathematical reality, and this

which should be only relative if the seconds are to participate in the firsts –, about the aporias of Zeno on space –, the kinds of being in *Sophist*, the composition of the limited – *peiras* – and of the illimited – *apeiron* – in *Philebus*, etc.), the Platonist dialectics suffers “metamorphoses”: cf. Dixsaut 2001.

¹⁸
Kitcher 1984.

¹⁹ Mouloud 1989. I develop this notion more in depth in: Barot 2009b, p. 167-179.

is the historicity of its objectivity which indicates it.

(iii) This problem of legitimization is at the heart of all formalization attempts, because the latter activate, by definition, a functional principle of discrimination. Formalization, activating the demarcation between mathematics and non-mathematics by trying to filter the second in the canons of the first, is thus an effect and an agent of the “epistemological rupture”. But the will to formalization is not that much the moment where the difference between the conceptual and the formal would be manifested (the implicit idea being that it would already be established), as the moment where it comes into existence, is actualized. In other words:

(a) From a diachronic point of view, it is the very fact of mathematization, of the formalizing process, which retroactively institutes the division into philosophy and mathematics. What is specific about this inversion of temporality is that it naturally transforms, when the process works, contingency into necessity. Science is instituted and is legitimized by being fulfilled, that which gives to the mathematical practice its clear primacy: the mathematizing will therefore envelopes and reveals by its simple “fact” that it is very much more than a “fact”, it is the historical process of the structural complexity of objectivity.

(b) But from a synchronic, structural, point of view, it reveals the highly stratified character of the latter, to the effect that even in a formalized theory, especially because of the structural incompleteness which characterizes it, there is the conceptual which remains irreducible to the logico-formal (I started my text with this aspect of the problem), even if it is latent, and becomes patent only in the case of crisis.

Note: Objectivity, Historicity and Fetishism

Mathematization is always an initiative of selective legitimizing, and for this reason understanding its stakes cannot be effectuated in an ahistorical manner, we have already indicated this. More generally, there is no independence of rationality as construction of objectivity, no matter what its modes (logico-mathematical included) are with regard to historicity, that is to say forms of practical and theoretical sociality with which it is necessarily at one, and that they refract even if they are evidently not a mechanical “reflection”. That does not mean, flatly and

naively, that mathematics has a history but that it is not invalidated in spite of that: it means, on the contrary, that the question of the necessity of mathematical knowledge should be thought of along with its historicity, and the whole problem is to deal with the nature and the modality of this intersection.

The important point to keep in mind is that all evolutive knowledge of objects, including mathematics, possess a layered internal structure, combining logico-formal stratum, cognitive and socio-institutional constraints²⁰ in a broad sense, historical-conceptual determinations, the last two kinds of components being, strictly speaking, inassimilable into the first, that is to say partly irreducibly clandestine, and irreducible to formal unequivocalness. Yet these three layers are *a priori* in the sense that we cannot refrain from presenting them, but not *a priori* determinable if we look at the variations of their material contents which form the concrete becoming of the sciences: if there is a “transcendental”, it is only in a not “neo” but post-Kantian sense, and in reality anti-Kantian revised and corrected. These three layers can be said *a priori* in the sense that they are structurally present in all positive knowledge, but are not *a priori* determinable with regard to their content: just as the *a priori* of ontogenesis is the *a posteriori* of phylogenesis, or, in a more Kuhnian sense, the structural principles of a paradigm are the *a priori* of the instituted normal science, even if they are the *a posteriori* of history which has led to them. As for the moments of crisis, and especially revolutionary crisis, that is to say the transition between two paradigms, they are the moments where “enigmas” which had emerged in the midst of the first paradigm did not find their resolution in it, and which have consequently occasioned the interrogations to move from the “normal” and technical level to the speculative one. Consequence: all “fetishisms” of mathematical objectivity, all “realist” and ahistorical hypostases of this objectivity, should be renewed or affected by tension using this fundamental sociality of rationality, be it speculative or technical-formal.

20 Evidently, this tripartition is grossly schematic. I conjoin cognitive and socio-institutional foundations due to the fearsome difficulty to which dissociating, to say it rapidly, the “natural” (related to the cerebral and psychological complexion) and the “cultural” (and by that I mean the economical, the sociological, the strictly institutional, etc.) can give rise. This is the whole interest of a “dialectics of nature” as element in the midst of “science” / “dialectics” of history, as theory of the way in which all natural conditions are always or become a social condition by means of their human appropriation.

1. 2. Retouring to Dialectics

1. 2. 1. Shaving Technique of the Ontological Difference

I have invoked the fact that for Lautman, metaphysics deals with supra-historical, and not specifically mathematical, “Ideas-enigmas”, for which mathematical theories try to come up, in history, with solutions. As for the mathematical philosophy, it is a “dialectics” which examines, in both senses, this relationship between the supra-historical and the historical. We can think that the difficulties and the speculative character of this ontological difference between Ideas and theories should be abolished – which means that this difference should be abolished as ontological -, and many versions of this abolition have been defended.

(i) We can, for example, read Lautman as a materialist, similar to what Lenin did to Hegel, and turn this difference into a difference of practices, successive or coexistent: between a critical-reflexive practice (critical or at the foundational level, for example during periods of crisis of paradigm, or of revolution) and a practice of direct production (in the “normal” regime). Similarly, we can materialize more substantially Kuhn or Bachelard. One way to proceed is, for example, by dismissing the ontological difference as a psycho-cognitive product of the phylogenesis, that is by apprehending it, anthropologically, as a sign of the age-old relationship of social individuals to a world that they try to control but which always evades them in one way or another – and from this point of view, the identification of the Piagetian spirit of the operational schemes, that is the progressive structuration of the perception, could be considered as a fundamental layer of all reflexive structuration of the real of which mathematics would only be the most rationalized version²¹. Different tastes can find different forms of this more or less reductionist materialization appealing.

(ii) We can also try to reduce the difference by doing away its ontological character without materializing it, for example by explicitly reintegrating the Ideas in mathematics: the most radical way of proceeding in such a way is by simply mathematizing them. This is where the question of the formalizations of dialectics makes its entrance. Concerning Lautman, the essential point to keep in mind²² is that we note two astonishing reversals of history. The first one is that Lautman identifies mathematics with dialectics because

21 Cf. I had proposed elements in this direction in Barot 2002, p. 33-72.

22 I develop that in Barot 2010, pp. 128-129.

of the Gödelian prohibition: this mathematization does, therefore, the exact opposite; it leads to a distinctly anti-Lautmanian way of proceeding, even if Lautman seems to have been its initiator. The second one is the following: we moved from the question “can we formalize a concept?” to the irreducible hiatus between dialectics and mathematics by means of which the question, through the problems raised by it, finds an adequate formulation. And yet, the idea of a formalization of dialectics has to take on two challenges: on the one hand, it completely illustrates this problematic dimension, but on the other, whereas dialectics since Plato to Lautman was a way of posing the problem of the relationship between the conceptual and the formal, as one way of treating this problem, it becomes, in its turn, its object. Let us get back to Hegel now.

1. 2. 2. Brief Reminder of the Hegelian “Spirit of Contradiction”

The 17th century marks the beginning of a conquering and complete rationalization of the objects and the forms of knowledge under the seal of the operational mathematization, inaugurating a gigantic effort of absorbing, modeled on the insights of physics and mathematics, and in the name of Reason, the “speculative” into the “positive”. This desire to reduce the “non-positive”, the inexact, the metaphysically suspect, the obscure, the confused and the obscurantist (and most particularly the religious) has been vital for the social, economical and cultural transformation of feudalism. However, this reshaping of rationality was constitutively effectuated with the aid of an operational form and language of reason towards nature and humans, which nourished an increasingly instrumentalist representation of the real and of reason itself. To put it more abruptly: this reshaping since the 17th century has led, in part, to a mutilation of the complexity of the real and its thought, and it is against this mutilation that Hegel rose up during the first third of the 19th century: refusal of the absorption of the speculative into the positive, of the reduction of the qualitative to the quantitative, of thinking to calculation.

Hegel rejects the thesis based on which the reality can be dissolved in the positivity, and the speculative in the scientific. In the real, we also have the workings of the possible, and history is always, unless we believe that it is written in advance, the realization of certain possibilities. This reveals that there is an undetermined aspect in the historical real, and that should be duly cleared up. And yet the world is

one: this non-being can only emerge from within being itself. The only possibility is therefore that being is in tension, in contradiction with itself. Moreover, for Hegel (1) thinking should grant and explain that, but even more (2) it should take itself into account whilst doing that. And this thinking being a dimension of reality, it should consider that it is equally affected, itself, by this negativity. Consequently, separating abstractly the form and the content of thinking is not thinkable: each form is a certain content's form.

Whence his critique of the formalism of Understanding in *Science of Logic*: not as a modality of rationality, but a modality imposing itself as the unique model of rationality, that is as pretending to be exclusive and hypostasizing its form, in this particular case deductivist and calculationist, appropriate to positive knowledge, by transforming it, in a royal manner, into the form of thinking in general.

1. 2. 3. Attraction, Repulsion: Suspicions

Formalizing dialectics has however had an important sense in a number of trends related to Marxism, because it seemed that such formalizations would lead to an additional legitimization of the dialectical scientificity. And yet, and it is understandable, this operation generates with itself attraction and repulsion. On the one hand attraction because it is, after all, a very fascinating operation, all the more so because its stakes - linking together science and philosophy, and sketching perspectives about novel redeployments of their relationships -, are high. Repulsion on the other, and on occasion giving rise to a double suspicion. (1) Dialectics has always been thought (Hegel, Marx, Marcuse, Sartre ...) as an alternative to all forms of logicism, and as against the form/content separation which is constitutive of all formalisms. (2) On the other hand and correlatively, this is why dialectics, and its subversive core (the existence of crippling, driving, fertile contradictions, etc.) have always been accused, from the perspective of the positive sciences and the formalist logician, of irrationalism. There exists, therefore, a suspicion coming from both camps, based on the irreducibility of the presumed antagonism of the two forms of rationality. Thus formalization is here presented, above all, as a will to reduce the doubly problematic antagonism. (1) From the scientific camp's point of view, that appears as a praiseworthy effort for reorienting the lost sheep (the dialectician): but then the only really good thing in the operation is the formalism itself, and not the dialectics that the latter pretends to capture. This would finally attest, retrospectively and at best, to the scientific, logico-deductive uselessness of the operation (as it was actively repeated by Granger),

and therefore of this presumed irrationality. (2) On the other hand, in the dialectician's camp, that can then appear as a form of betrayal or treachery: pretending to formalize the unformalizable, institutionalizing the uninstitutionalizable, is that not conjointly opportunism and revisionism? Vain or useless effort on one side, betrayal on the other, the idea seems to be that we do not reduce the irreducible (accused of irrationalism in one case, of over-rationalism in the other) by moulding it in the canons of the operational rationality, if not by abolishing it as such – that is, in both cases, by making contradiction disappear. Finally, we get from there to the following situation: there exists indeed a profound hiatus between dialectics and mathematics, and this hiatus is primordial: taking dialectics seriously implies that we recognize that its formalizations miss it. And yet, the question of its legitimacy is therefore the first question, and it is a circular one, because it is only by posing its legitimacy that we can certify it or justify it.

Here, the entirely political dimension of the problem emerges. We find ourselves, in effect, in a limit-case of the relationships between the conceptual and the formal which is by no means insignificant: the negative thinking, discourse of internal contradictions and of the movement of the possible in the middle of what is, discourse which examines what is by using the vocabulary of that which is not, Marcuse has shaped it for us, is the condition of possibility of all revolutionary perspectives. For Lautman, the connection between metaphysics and mathematics is not contingent but necessary: just as here I affirm that these attempts at formalizing dialectics link mathematics and politics in a necessary manner. I will come back to this point in section 4.

2. Paraconsistency According to Graham Priest

There are two major starting points in the approach of Priest, a major contemporary theoretician of the paraconsistency. The existence of a continued interrogation about the paradoxes that affect the argumentative discourse because of the self-referentiality of certain affirmations or reasoning²³ (such as “the liar paradox”) on the one hand, Gödel's 1931 theorem²⁴ which demonstrates that if mathematics is consistent, it is incapable of demonstrating all the truths that it is nonetheless capable of constructing on the other. Gödel's procedure consists of making a detour by a “metalanguage” L' in relation to a “language” L (arithmetic suffices) in which we exercise our naïve capacity

23 Cf. Priest 1979

24 Priest 2006a, pp. 39-50.

of proof. If a statement P of L is not provable in L, we can code it then turn this code into the object of demonstrations in the metalanguage M, demonstrations which appeal to the notions of truth, its properties, its relation to the provability, etc. We can, from that point, find in M a proof, this time of P. Gödel constructs in L the statement “if arithmetic is consistent, then it is incomplete”. He codes this obviously self-referential statement in the metalanguage L', and demonstrates it: to say it within the terminology of the problem that here interests us, thus the theorem demonstrates an undecidability, it proves an improvable. It is a theorem of limitation, which indirectly attests to the fact that self-reference carries with it paradoxes²⁵. Priest²⁶ undertakes a rereading of this Gödelian incompleteness and its effects by means of a discussion of the approach proposed by Tarski in 1933-1935²⁷.

2. 1. (In)consistency, (In)completeness and Semantic Closure: From Gödel to Tarski

We have already seen the way Lautman drew from the Gödelian incompleteness the necessity of assimilating again, against the spirit of the Hilbertian foundational program whose impossibility is attested to by the 1931 theorem, metamathematics into dialectics, and of seeing and comprehending the historical work of the hiatus dialectics-mathematics by means of Plato and, marginally, of Heidegger. The 1933-1935 theory of Tarski can be read, inversely, like a solution of technical bypassing of the problems coming out of the incompleteness. The 1931 result relies on the hypothesis of the consistence of the considered formal systems, and shows that their incompleteness is implied by this consistence. And yet, the important point is that incompleteness exists only if we want it to exist – semantic version of the limitation – that the systems under consideration totally control their semantics (which means that they are capable of proving with regard to their proper “truths”). We can then say, by contraposition, that in Gödel, if consistence implies incompleteness, then completeness implies inconsistency. And for Tarski, the problem is very much there: it is necessary to avoid inconsistency. Therefore, it is necessary to attack its origin: “completeness”.

And yet, all completeness can only rely on a “semantic closure”. The semantic closure of a language (of a theory, of a formal system,

25 Cf. Cassou-Nogues 2004, ch. III.

26 The most systematic presentation of Priest's approach can be found in Priest 2006a.

27 The important text is the monograph from 1933-1935 “The concept of truth in the deductive sciences”. I rely here on his own synthesis Tarski 2009, pp. 247-277.

stated in this language), for example natural language, relies on two elements: it contains the “names” of its proper “statements”, and is capable of defining what does the fact of “being true” signify for these statements. To take up again the example of Tarski, “snow is white” is a name of the statement “snow is white”. Here the principle is that to all statement α , we can associate a name $\underline{\alpha}$. The important relation between α and $\underline{\alpha}$. Tarski tries to fix what does it mean, for α , to be “true” and he effectuates this by means of the following equivalence:

$\underline{\alpha}$ is true if and only if α

If “T(x)” means “x is true”, then we can rewrite the equivalence in this way:

$T(\underline{\alpha}) \Leftrightarrow \alpha$

This is the “schema-T” or convention-T of Tarski, by which the latter fixes the criterion of “material adequacy” of a statement to the real, based on a very classical approach, correspondence, that is to say according to which a statement is true if it corresponds to what is the case. The problem of a semantically closed language is that it is capable, like natural language, to produce statements about its own statements: it is self-referential. And this self-referentiality engenders paradoxes, contradictions, that is to say it moves us away from a truth which can only result from a coherence of the real itself. This is why Tarski, searching to avoid paradoxes, eliminates the conditions of possibility of self-referentiality by rejecting semantic closure. In order to do so, it is necessary to establish at least one duality between the language L, and the “metalanguage” L' within which we will be able to say if such or such a statement of L is true or not. Naturally, this duality is a logical operation: it is not that much the capacity of constructing distinct languages that counts, but being able to produce a strict demarcation, if we work in a given language, between the latter taken as “metalanguage”, and a part of it which will be “language-object”. One of the consequences and difficulties of this affair is that it leads to fix outside of language-object its truth predicate.

To sum up, given that it is most important to avoid paradoxes, it is necessary to reject semantic closure (responsible for inconsistency), without this leading to incompleteness, and establishing a truth schema, the convention-T, within the spirit of the Gödelian coding procedure ensuring the move between language and metalanguage, allows to rely on the conditions in which metalanguage can say the truth of language whose metalanguage it is.

2.2. Under-Determination of the Tarskian T-schema and Orientation Towards a Trivalent Semantics

A proof is a process by which we establish that an affirmation is true: disposing an affirmation, we try to find either its proof or its refutation. But based on what? Based on other affirmations which are true, i.e. for which we have already provided proofs etc., and this can give rise to an infinite regression, or to undecidable statements (neither demonstrable nor refutable). Many paradoxes are born in L due to self-reference, and Tarski's solution consists of distinguishing between L and the metalanguage L' which is more powerful than L and contains it. For Priest, for whom natural language remain the primordial concern, the particular case $L = L'$, a case where L is sufficiently powerful for treating its own semantics, remains the most important one: he radically defends the principle of semantic closure²⁸. And if we have this equality, then the paradoxes will reemerge: but that poses a problem only if we want to avoid paradox. The originality of the paraconsistent approach is located at that point: for Priest, the goal is not to suppress paradox, but to put up with it for two reasons. (i) Most of the discursive operations are not paradoxical, (ii) at the other end of the problem, due to the fact that reality itself is that which leads to semantic paradoxes. In short, it is useless to dramatize, because in the first case it is not dramatic, the problem is marginal, and in the second case it is inevitable. In a word, a marginal problem inevitably exists. Whence the fact that Priest is essentially interested in these self-referential situations where $L = L'$, that is in semantically closed L theories. For him, it is necessary to accept that the correct formalization of our methods of naive proof is a semantically closed theory containing semantic paradoxes, that is to say a theory at a certain level “inconsistent”. His goal is of course to show that at a certain level and under certain forms, “inconsistency” does not carry with it irrationality, is not outside of logic. His aim therefore consists of keeping the T-schema, all by imposing a new semantic signification on it.

Priest hence vigorously challenges the Aristotelian perspective²⁹, and intends to provide the conceptual foundations of this refusal, that is to say, to contradict Aristotle on the latter's own foundational ground: the ontological one. The goal, henceforward, is not to avoid “inconsistency” but to let it have currency all by isolating it, to give a direction, a local existence to it, so that it does not put a strain on the system globally. Whence his first strong thesis: the consistency hypothesis, which is the

28 Priest 2006a, pp. 125-140.

29 Priest 2006b, pp. 7-42.

first principle of the 1931 theorem, should be rejected³⁰. But the second one is equally important: inconsistencies emerging from the semantically closed systems which are considered should be logically characterized as rational configurations, hence rationalizable.

I mentioned in section 0 that all logical theories of negation are in reality always visions of contradictions, of the relation of contradiction. The main implication (if, for example, we distinguish, like Da Costa, between formal, semiotic and real contradictions), therefore, of Priest's theses is that, because semantic paradoxes are not outside of reason, it is necessary to be able to distinguish between exclusively true affirmations, exclusively false affirmations (that is to say "classical" affirmations) and affirmations which are simultaneously true and false, that is paradoxical. This distinction consists of saying that an affirmation can accordingly take three distinct truth-values: true, false, and paradoxical. At the semantic level, the Priestian paraconsistency is therefore translated by the rejection of strict bivalence, in favor, prototypically, of trivalence {T, F, P}. It is by exposing this semantics in "The Logic of Paradox"³¹ in 1979 that he began his works, the matrix of the paraconsistent semantics that he would later on develop.

Two things should be now clarified. (1) What is the formal, syntactic and deductive structure adapted to such a semantics? (2) What is the link between such a semantics and Priest's ontological bias? Let us deal with these two points in order.

2.3. Principle of the Formalism of Paraconsistency

Here, I allow myself to use Da Costa's account³², but with two correlative biases: on the one hand, I will only use a vision of his general architecture, by leaving those aspects of it which are not directly related to my aim aside, and on the other, I presume that this vision, without trying to evaluate the exactitude of the operation in detail, suits Priest. The essential point is to make their general approach clear, and Da Costa is clearer in his account: the style of formalism that he mobilizes, the sequent formalism, is totally adequate for this general aim. The base unit, the sequent (from the Latin *sequor*, "to follow from") is composed of two

30 He makes the distinction between the law of non-contradiction $\sim(A \sqcap \sim A)$ and the "principle of consistency" based on which no affirmation is simultaneously T and F: for Priest, that $\sim(A \sqcap \sim A)$ be true cannot prevent, by itself, the instances A and $\sim A$ from being true. Given that it is the hypothesis of consistency that implies incompleteness, rejecting it (accepting the conjunction $A \sqcap \sim A$ to be T), opens up important perspectives.

31 Priest 1979.

32 Da Costa 1997, p. 237 and suiv.

collections of written formulas on the left and on the right of a symbol « $\vdash$ » which signifies "proof":

$$A, B, C \vdash D, E, F$$

When we have a sequent without formula(s) on the left, of the kind

$$\vdash P$$

it is that the formula P is deduced without us requiring any hypothesis: it is a formula which is "true" according to its own logical form³³. As for Priest's "**dialetheia**", or dialectical contradiction, it is therefore a statement in the form of:

$$A \wedge \sim A$$

Where, we should remember, $\sim$ is a symbol of negation, and " $\sqcap$ " the symbol of conjunction ("and"). What is unique about paraconsistent logics is that they establish that we can, for a given theory T, and without this causing any harm, affirm

$$T \vdash A \wedge \sim A$$

And yet, when it comes to the principle « EX CONTRADICTIONE SEQUITUR QUODLIBET » (called EC from now on), based on which the presence of a contradiction in a theory renders it trivial, in other words allows to derive anything from it, classical logics are all in agreement. Let A and B be formulas of T:

$$\forall A, \forall B \quad A, \neg A \vdash B$$

And yet, this "triviality", the fact that we can demonstrate anything, makes us lose all kinds of rationality, all interests for the system. The paraconsistent logician does agree: what he then needs is to establish that we can have $T \vdash A \wedge \sim A$, without T becoming trivial. This condition

33 It is important to distinguish between P as "tautology" from the semantic point of view, which is written $\models P$, and P as "theorem" from the syntactical point of view, which is written $\vdash P$. Here I do not go into details of the formal systems which contravene the completeness theorem based on which semantics and syntax coincide, i.e. such that: $\vdash P \sqcap \models P$. Obviously, it is when this coincidence disappears (cf. Gödel 1931) that not only things become interesting, but especially the problems at the origin of paraconsistency emerge. In this sub-section, I am only interested in the general technical principle.

of non-triviality should be simply translated by the fact that, given any two formulas of T,

$$\forall A, \forall B \quad A, \neg A \vdash B$$

A paraconsistent logic is not trivial because by this definition B is not tautological, or, at the syntactical level, theorem. How do we proceed? By a particular interpretation of negation in the subformula “ $\neg A$ ” of the last writing above. Let us fix, in advance, that the negation with which paraconsistency works is “ $\sim$ ”, which is different from the classical negation “ $\neg$ ”. Whence the two following questions: on what relies the definition of “ $\sim$ ”? What is its relation to “ $\neg$ ”?

Relatively to what interests us here, let us remark that only one rule suffices to define, indirectly, the classical negation: it is simply *reductio ad absurdum* (reduction to absurdity), the definition which takes us back to EC.

$$\frac{\frac{G, \neg A \vdash B \quad H, \neg A \vdash \neg B}{\text{Reduction to absurdity (EC)}}}{G, H \vdash A} \quad (\text{RA})$$

Or, more intuitively:

$$\frac{G \vdash A, B \quad H \vdash A, \neg B}{G, H \vdash A}$$

N. Da Costa weakens RA by adding on the right an additional condition, and he names RA_1 the obtained result:

$$\frac{G \vdash A, B \quad H \vdash A, \neg B \quad I \vdash \neg(B \wedge \neg B)}{G, H, I \vdash A} \quad (RA_1)$$

With the system of classical natural deduction NA, {NA, RA_1 } is very weak; we cannot derive the excluded middle. We can then have this last one as axiom:

$$\vdash A \vee \neg A \quad (\text{TE})$$

{NA, RA_1 , TE} is named *C1* by Da Costa, non-trivial paraconsistent system, which is used by him as matrix. He then posits:

$$\neg^* A \Leftrightarrow \{\neg A \wedge \neg(A \wedge \neg A)\}$$

Then fixes:

$$\frac{G, \neg^* A \vdash B \quad H, \neg^* A \vdash \neg^* B}{G, H \vdash A} \quad (RA^*_1)$$

That is to say:

$$\frac{\frac{G, \neg A \wedge \neg(A \wedge \neg A) \vdash B \quad H, \neg A \wedge \neg(A \wedge \neg A) \vdash \neg B \wedge \neg(B \wedge \neg B)}{G, H \vdash A}}$$

Let:

$$\frac{G, \neg(A \wedge \neg A) \vdash A, B \quad H, \neg(A \wedge \neg A) \vdash A, \neg B \wedge \neg(B \wedge \neg B)}{G, H \vdash A}$$

That amount to forging “ $\neg^*$ ” as a stronger version, more constraining, of negation, in the classical negation’s terms. The operation then consists– it seems to me – of somewhat translating this definition, by saying that “ $\neg$ ” is the classical negation, and “ $\neg^*$ ” a weak negation in the terms within which this classical negation is defined. Whence this definition and this rewriting:

DEFINITION. Let the unary connective $\sim$ defined by:

$$\neg A \Leftrightarrow \sim A \wedge \sim(A \wedge \sim A)$$

The classical negation $\neg$ is thereby defined within the terms of weak negation $\sim$, which is the proper paraconsistent negation. Then we have:

$$\frac{G, \neg A \vdash B \quad H, \neg A \vdash \sim B \wedge \sim(B \wedge \sim B)}{G, H \vdash A} \quad (RA^*_1)$$

On the other hand, without the third condition (dotted box) or RA^*_1 , we then have:

$$\frac{G \vdash B \quad H \vdash \sim B}{G, H \vdash B \wedge \sim B} \text{ (introduction } \wedge \text{)}$$

The EC cannot be applied, $B \wedge \sim B$ is not valid. Q.E.D.³⁴

$B \wedge \sim B$ is the paraconsistent contradiction or “dialetheia”, deductible based on standard logic, which naturally provides the framework of paraconsistent logic. It is, in effect, based on the former that the latter, via the above definition, institutes a “weak” negation that allows it to satisfy the existence of contradictions. The principle, furthermore, is equally “classical” in the traditional sense: it is the relativization of the principle of non-contradiction’s scope. But, given that the classical negation is defined as combination of formulas and occurrences of conjunction and of the paraconsistent negation, the relationship between these two negations leads to read the process as a move which turns the classical negation into a derivative of the paraconsistent negation – which turns the “strong” into a combination of occurrences of the “weak”. These inversed modes of hierarchization of the classical and the non-classical would merit immense developments, but I do not pursue it here.

Let us now move on to point (2), that is to say to the question of the relationship between semantics and ontology.

2.4. Semantic Affairs of the Priestian T-schema

For Priest, with a classical semantics (a set model for example), we can effectively characterize the sense of a statement and the conditions in which it is true. But, on the other hand, that does not give us the

³⁴ Priest adds, however, in Priest 2006b, § 4.8, p. 86, that his dialetheism can leave RA unaffected under a certain angle. When $A \Rightarrow (B \wedge \neg B)$, the classical use of RA consists of inferring $\neg A$: by contraposition we draw $\neg (B \wedge \neg B) \Rightarrow \neg A$, and by De Morgan and elimination of the implication, $(B \vee \neg B) \Rightarrow \neg A$. Priest says that the classical sense of RA consists not of establishing something, but of forcing an enemy to abandon his affirmation of A. From the dialetheical point of view, on the contrary, RA is not logically sufficient for that, because that presupposes that the law of excluded middle (that intuitionistic logic equally contests). In other words, classical RA constitutes the bivalence: the problem of RA is not the principle of the reduction that it operates, but its fundamental sense, which remains the ontological presupposition of Aristotle. All that for saying that the syntactical solution of paraconsistency, the weakening of RA, is a consequence or an effect of a prior decision. Priest is very laconic in this § 4.8, but it seems to me that, essentially, his intention is to remind that the fundamental problem is not a technical one, but very much ontological, something that I find just. It is on that ground that he concludes, taking care to add that if a contradiction is logically possible it is not necessarily rational to believe in it, i.e. believing that all logical contradictions have an effective counterpart in the real.

meaning, for this statement, of being true, and especially, such a classical semantics does not avoid the semantic “jumps” between the purely true and the purely false, jumps which are induced, by definition, by the dialetheias of this form $B \wedge \sim B$. Whence, we have mentioned it, the move to a trivalent semantics, and in particular the distinction which reformulates the idea of “paradox”, between Untruth and Falsity³⁵. Avoiding jumps or semantic gaps forces us to refuse the classical assimilation of the untrue in the false: the “untrue” is here the “paradoxical”, something that is neither simply (purely) true, nor simply (purely) false. Let us see the way it operates.

At the level of the institution of semantics³⁶, the Priestian approach consists of linking together the formalism briefly sketched above and the T-schema of Tarski. It should be reminded that truth predicate allows us to postulate the equivalence between the affirmation of the truth of a phrase $\underline{\alpha}$ and the affirmative statement α of a state of affairs of which this phrase is the translation.

We can eliminate the parentheses for lightening the writing³⁷. What needs to be determined is, therefore, by substitution, under what conditions and with which meaning we can have

$$T(\underline{B \wedge \sim B}) \Leftrightarrow B \wedge \sim B$$

In order to make clear the conditions in which the truth predicate T can adequately characterize this conjunction³⁸, Priest characterizes,

³⁵ Cf. Priest 2006b, p. 69 and suiv.

³⁶ The presentation occupies above all Priest 2001, p. 53 and suiv., starting from § 4.2 on the “The T-schema”.

³⁷ Moreover, by means of a coding *à la Gödel*, Priest sometimes treats the equivalence $T(\underline{\alpha}) \Leftrightarrow \alpha'$, where α' is the *code* from α , $\underline{\alpha}$ then being the name/sentence of a coded statement. But this is secondary for our purpose here.

³⁸ Ibid., § 4.8 and 4.9, p. 67-72 then chap. 5 “Dialethic Semantics for Extensionnal Connectives”, pp. 73-81 for the principal presentation of these conditions. I am not saying, by any means, that Priest “tries to give the semantics of the deductive scheme of De Costa”. The latter proposes, moreover, for his system C1, in Da Costa 1997, p. 244-246, a bivalent non verifunctional semantics which inherits the non-verifunctionality of the weak negation $\sim$, that is from the fact that knowing the truth value of a formula A does not suffice to mechanically determine that of $\sim A$. Even if that amounts, in part, to “dualizing” the idea of negation (because $\sim A$ is not necessarily determinable in an univocal way), and to having the capacity to attribute simultaneously the values T and F to certain singular formulas, we are not, in spite of that, dealing with a trivalent semantics, the latter, as Priest does it, institutes a third possible truth value for these singular formulas. But this difference *stricto sensu* does not prevent the compatibility *lato sensu* of the two semantic approaches, therefore the legitimacy of a presentation of the Priestian semantics with regard to the De Costain architecture. I have clarified at the beginning of § 3 the didactic sense of this free “combination”.

firstly, the conditions of satisfaction of the principal connectors in a very traditional manner³⁹, in a recursive manner on an axiomatic basis. The first axiom, which is used as definition, is the T-schema of an atomic formula:

$$T\alpha \Leftrightarrow \alpha$$

He then defines conjunction and disjunction:

$$T\alpha \text{ and } T\beta \Leftrightarrow T\alpha \wedge \beta$$

$$T\alpha \text{ or } T\beta \Leftrightarrow T\alpha \vee \beta$$

And finally negation:

$$\text{It is not the case that } \alpha \text{ (i.e. } \neg\alpha) \Leftrightarrow T\neg\alpha$$

The question is: what does “it is not the case that” mean? That is to say, what does “ $\neg$ ” mean? Can we directly infer, from the fact that nothing in the world certifies that α is true, that α is false? In order to understand Priest’s responses to these questions, it is necessary to translate “it is not the case that α ” by the equivalence $T\neg\alpha \Leftrightarrow \neg\alpha$, as he does it, that is to say postulating $T\neg\alpha \Leftrightarrow \neg\alpha$ as the T-schema for negation already contains its interpretation of the meaning of “ $\neg\alpha$ ”, that is to say his thesis. Indeed, two possible solutions exist in reality, which he makes explicit a little bit further⁴⁰:

Let $\neg\alpha$ mean $T\neg\alpha$.

Let $\neg\alpha$ mean $\neg T\alpha$.

Classical logic assimilates the affirmation based on which *it is true that it is not the case that α* ($T\neg\alpha$) to *it is not true that it be the case that α* ($\neg T\alpha$), both are referred to a same affirmation of the falsity of α . In other words, the classical approach⁴¹ implicitly states that

$$T\neg\alpha \Leftrightarrow \neg T\alpha$$

then states, F being the predicate of falsity, that $T\neg\alpha \Leftrightarrow \neg T\alpha \Leftrightarrow F\alpha$.

39 Ibid., p. 60.

40 Priest tends to distill the steps of his approach between diverse digressions; Here, I will therefore content myself with trying a reasonable reconstruction of his proceeding.

41 Cf. Ibid., p. 64.

And yet, the affirmation $T\neg\alpha \Leftrightarrow \neg T\alpha$ is a biconditional, that is to say the conjunction of an implication and its converse⁴². Priest is attentive to distinguishing them:

$$(1) T\neg\alpha \Leftrightarrow \neg T\alpha$$

$$(2) T\alpha \Leftrightarrow T\neg\alpha$$

Given that by the *T-schema*, we have, for the negation $T\neg\alpha \Leftrightarrow \neg\alpha$, **in case (1), that implies:**

$$(\neg\alpha \Leftrightarrow) T\neg\alpha \Leftrightarrow \neg T\alpha$$

(If it is not the case that α), it is true that α is F, therefore it is untrue that α be the case. The falsity of α implies its untruth.

In case (2), that implies

$$(\neg\alpha \Leftrightarrow) \neg T\alpha \Leftrightarrow T\neg\alpha$$

(If it is not the case that α), it is untrue that α be the case, therefore it is true that α is F. This time, the untruth of α implies its falsity.

Now, let us see what we will have if we have α and $\neg\alpha$, that is to say respectively by the T-schema $T\alpha$ et $T\neg\alpha$. Based on the conjunction’s definition, we then have

$$T\alpha \wedge \neg\alpha$$

that is to say by De Morgan

$$T\neg(\alpha \vee \neg\alpha)$$

By principle (1) ($T\neg\alpha \Leftrightarrow \neg T\alpha$), that gives

$$T\neg(\alpha \vee \neg\alpha) \Leftrightarrow \neg T(\alpha \vee \neg\alpha)$$

It is true that $\neg(\alpha \vee \neg\alpha)$, let $\neg(\alpha \vee \neg\alpha)$ be false, then $(\alpha \vee \neg\alpha)$ is untrue. And if $(\alpha \vee \neg\alpha)$ is untrue, then $(\alpha \vee \neg\alpha)$ is true, that is to say (by De Morgan) that

42 Ibid., p.70.

is true, because being true and untrue, it is not false.

We can therefore provide the semantic affirmation $T(B \wedge \sim B)$ for the syntactic deduction of $B \wedge \sim B$, which is what we were seeking. This amounts to saying, finally⁴³, that everything can be expressed in terms either of truth, or of untruth, and that the falsity of the strict sense is nothing but a part of the untruth.

And yet, if we consider, this time, principle (2) ($\neg T\alpha \chi T\neg\alpha$), we start straightaway from an interpretation of negation as being destined to indicate (through the transitory intermediary of untruth, as antecedent of the implication) falsity (into which all untruth is absorbed and dissolved). The second principle transforms the conjunction of $\neg T\alpha$ and $T\alpha$ into the affirmation that α is purely and simply (that is absolutely) T and F at the same time, that is to say absurd. The classical affirmation $T\neg\alpha \Leftrightarrow \neg T\alpha \Leftrightarrow F\alpha$ identifies, absolutely, untruth with falsity, and that relies on the conjoint affirmation of the two abovementioned principles (1) and (2). Priest names them, respectively, principle of exhaustion and principle of exclusion. The dialetheist totally accepts the required exhaustion, but rejects, on the other hand, this exclusion, which expresses nothing other than strict bivalence. Conclusion: principle (2) should be refused⁴⁴.

This amounts to saying that the biconditional of the T-schema, $T\neg\alpha \Leftrightarrow \neg\alpha$, should not be biconditionally used in order to characterize negation, in short, that it is not an authentic biconditional⁴⁵.

To sum up, if falsity implies untruth, untruth does not imply falsity: untruth is therefore the “paradoxical” intermediary between the purely T and the purely F. The lack of support for an affirmation does not suffice to logically affirm its falsity, or in other words, a merely logical argument is never sufficient for affirming falsity: the classical logician should provide a proof for the falsity’s effectiveness, that is to say to exhibit something in support of the latter. Just as the dialethetician should provide an extra-logical proof for the fact that non-falsity is not a synonym for truth. In

43 Here, the classical interpretation of implication amounts to saying that the validity of implication does not hurt that of the antecedent, which is why Priest reassumes here one of the paradoxes of material implication invoked in section 0. The big difference is that he reassumes it explicitly on the basis of motives which lack in the classically treated material implication, all by stipulating, I will come back to it later, the limits within which this “paradoxicality” should be fitted.

44 Priest 2006a, p. 78-80.

45 All of *ibid.*, ch. 6, “Entailment” develops this problem. For more details, it should be systematically referred to.

that case, all real proofs, he says, are combinations of *a priori* (logical) elements and empirical elements⁴⁶, and it is precisely at the empirical level that “paradoxes” are observable. It is therefore necessary to get out of the formalized concept in order to go towards the only thing that can complete its insufficiency: the world.

And here we need to pose a question: refusing that untrue imply F is also refusing that the untruth of $B \wedge \sim B$ imply its falsity. But what is the nature of the affirmation of this untruth itself? Is it absolutely T, or itself paradoxical, that is to say true and untrue? Can we, should we, and how, verify whether it satisfies, itself, the truth predicate? To respond to that, it is necessary to move to a superior level of language, like in Tarski, moving to a metalanguage etc. If we remain at the level of language, that is within the semantic-syntactic level, it is infinite regress that therefore begins. The response to this question is impossible from this purely logical point of view, quite simply because for Priest, the logically admissible character of $B \wedge \sim B$ does not harm the rational character of believing in its reality: or in other words, it is necessary to discriminate, in the world, between what is contradictory and what is not.

Providing a semantics, a model, as plurivalent and alternative as it be, remains an intra-logico-mathematical operation. It results from this that the formal under-determination of the conceptual content of truth predicate is not compensated by an alternative semantics of this kind. The sense of statements and their truth are distinct things, he even says that they are “independent variables”⁴⁷: this amounts to saying, naturally, that logics is incapable of defining the truth. It is therefore necessary to nuance the Fregeian patronage previously invoked: if fixing the sense of a statement is giving its truth conditions, giving these truth conditions is not giving this truth itself. To this end, an ontological solution is required, and this is the case in Frege himself, who opts for a hyperrealist solution (in the sense of a “Platonist realism” of the logico-mathematical objects).

2.5. From Semantics to Ontology: Examples of “dialetheias”

But if, for Priest, it is necessary to go beyond not only syntax, but also semantics *stricto sensu*, because there is a conceptual under-determination of the truth predicate, in short, if the situation imposes an ontology, it is the radically anti-Fregeian path that he takes. In so doing, he finds again the fundamental theme of Lautman for whom the

46 *Ibid.*, p. 67.

47 *Ibid.*, p. 60.

rapprochement between mathematics and philosophy is necessary, because the objet of mathematics is irreducible to its objects (syntactically-semantically determined), and this theme is translated by a same refusal of all logico-mathematical ontologies. In short, like Lautman he avoids what I call the ontological pitfall (which carries a generic fetishism), that is to say the belief based on which the ontological problem of logic and mathematics is a logico-mathematical problem, which would necessitate an oscillating position between the “realist” pole and the “nominalist” pole⁴⁸. The sense of the approach is fundamental: it consists of refusing to locate the ontology of or concerning logico-mathematical within logico-mathematical, be it for assuming it or criticizing it. In short, the approach radically displaces the problem’s ground. It is true that Lautman locates the ontological problem in the Ideas by etherifying it, and we have already mentioned the fact that the “ontological difference” could be dealt with in ways more convincing than the way he thought it. Priest, in my eyes, takes such a path: he situates the ontological problem in the concrete reality, anchors, in a Hegelio-Marxist⁴⁹ mode, objectivity to a reality defined by the fact that it is the condition and the object of concrete practices⁵⁰.

Let us get back to the first question: why avoiding inconsistency is not the goal? Why is it appropriate “to accommodate them”⁵¹ only, or as Da Costa puts it, to “master and control”⁵² contradictions? Quite simply because contradictions really exist. An authentically paradoxical affirmation, for Priest and for Da Costa, is the discursive expression of a paradoxical reality, or rather, of portions of paradoxical reality. This last nuance is important: “it is important not to multiply contradictions beyond what is necessary”⁵³ he says, an economical postulate which consists of saying

48 Naturally, one nominalist option is more directly favored by Marxists, it is “realism” which is the most radical mystification: here I agree with Priest 2006a, p. 151, who refers, again, to Marx on this point. But if nominalism is employed as an anti-ontological position on ontology’s ground (logico-mathematical), it does not elude this critique.

49 Priest 2006a, § 10.4 “Mathematical Realism”, p. 151.

50 Ibid., § 10.5, “... And Anti-Realism”, p. 153. It is here the point of fundamental articulation with the other angle of assault presented in Annex 2: its detailed articulation will be the subject matter of the next work on these questions. In what follows, I leave the systematic evaluation of Priest’s Hegelio-Marxist claim in suspense. The last part can be regarded, that said, as the indication of a limit of his approach (the way it appears in his texts): the absence of politicization of the stakes and of his ontology, and of the dialetheical edifice that he constructs on its basis.

51 Ibid., p. 72.

52 N. Da Costa 1997, p. 237.

53 Priest 2006a, p. 71.

that the world is not only filled with contradictions and paradoxes, even if it does contain a few.

The stake of the semantic closure is here manifested: it is because there is only one reality that there should be, basically, only one language, and this is why the latter is closed, and that there are paradoxes. The foundation of paraconsistency and of its semantics is an ontological monism. Priest takes up the Tarskian distinction between statement and name, in the form of the distinction between statement and its sentence, and the truth predicate “ $T(x)$ ”, by positioning himself under the authority, beyond Tarski, of Frege for whom giving the sense of a sentence is giving its truth conditions. And yet, a first objection that he addresses to Tarski is that his truth schema, if it characterizes (possibly) what it means for such a statement to be true, it does not provides a concept of truth, that is to say it produces a problematic semantic indetermination. As for him, he wants, on the contrary, to furnish such a concept of truth. For him “dialetheism”, his conception based on which true and logically receivable contradictions exist, does not, like standard logic, summon by itself, i.e. as a theory of logic, a particular conception of truth. All particular conceptions of truth presuppose, *de facto*, an otology, monist in his eyes.

Let us take a look, now, at the essential characters of this monism of Priest. He starts from the Hegelian affirmation based on which, in keeping with Kant’s “transcendental dialectics”, correct reasoning, proceeding based on the legitimate application of certain concepts, leads to contradictions: these concepts are therefore contradictory or carry contradictions. Priest takes up this idea: our concepts are inconsistent, they produce *dialetheias*. Hegel was therefore right, and logical paradoxes, whether semantic or set theoretical – with their common self-reference – bear witness to it, even if only by their appearances. But inconsistency does not imply incoherence, especially because this inconsistency happens, beyond discourse, in the real – that the real is never incoherent in the sense of being irrational. A therefore fully intelligible postulate.

Many non-literal interpretations of “contradiction” have been given, including, starting from the 1950s, those by Soviet philosophers, who defended the idea that contradiction, if it could belong to thinking, was not nonetheless real. But the general opinion, even when it satisfies the idea, does it by saying that dialectical logic should be at least compatible with the Frege-Russell paradigm, the most general axiology for the norms of scientific and correct thinking. In that case, also, contradictions have often been reinterpreted in a softer fashion. For Priest, this Fregeo-Russellian paradigm is only a theory; on the other hand, and

this is a second very strong thesis, the central theoretical meaning of contradiction in Hegel and Marx is precisely the logical meaning⁵⁴.

2.5. 1. Movement

In two very interesting articles in the 1980s, Priest develops many interesting examples. The first one, that he takes up again in *In Contradiction*⁵⁵, is about movement (as the relationship between matter, time and space), and in particular, as Zeno’s aporias showed it in their time, continuous movement. In any concrete continuum, there exist either contiguous and opposed properties, that is to say a part of continuum where it is not true that all be A or not-A (for example, in a color continuum going from red to another color, there is an intermediary moment where we are still in red and outside of it), or a region where something is more simply A and non-A. Here we find movement again, in its generality, the way Hegel conceptualizes it.

The domain of classical logic is “consistent”, that is static. And yet, it is of course movement that engenders contradictions. Let C be a body situated in s. What is the difference that we can establish, at a given instantaneous moment, between C when its being is in movement, which by definition is not an internal state but a relational situation, and its being at rest? In a Hegelian fashion, let us consider the sentence A “C is in s”:

if C is at rest, A is true
if C is in movement, it has always already started to leave s:
therefore the negation of A is true.

Thus, A is true and false at the same time, and Priest’s goal is to supply a rigorous semantics for this affirmation.

2.5. 2. Alienated Work, Commodity

Priest also takes two examples directly from Marx: alienated labor in *The Economic and Philosophic Manuscripts of 1844* and commodity (use value/ exchange value) in *Capital*⁵⁶. He reminds us that in the *1844 Manuscripts*, human, for Marx, is telos, the generic self-development of the individual and of humanity by labor. And yet, alienated work is self-alienation,

54 “Priest 1989, pp. 388-415, p. 391.

55 Priest 2006q, ch. XI & XII, p. 159 and suiv., p. 172 and suiv., and a little bit further, ch. XV ; “III : Time”, p. 213 and suiv.

56 “Priest 1989, p. 398.

92 “To be and not to be – that is the answer”...

alienation of work by capital (dead labor, accumulated labor), that is the loss of essence. Like the self-realization of humanity, the work *h* is such that:

$$h = h$$

As alienated, the work *h* is however

$$h \neq h$$

Thus work is simultaneously identical with and opposed to itself.

Concerning the first volume of *Capital*, Priest looks at the commodity as an object *a* which can be used (fact *Ua*) or exchanged (fact *Va*). When *a* is used, it is not exchanged, and reciprocally⁵⁷. With ~ as the symbol of negation and ^ as the symbol of conjunction, that gives us the following:

$$\sim(Ua \wedge Va)$$

However, in commodity exchange, each commodity is linked to another one as *Va* and as *Ua* at the same time, and especially the strong idea of Marx, exchange value presupposes its “carrier” use value, even if the latter is put in parenthesis from the point of view of capital’s accumulation. So that we also al-ways have:

$$Ua \wedge Va$$

And Priest adds in the article that the real “being” of *a*, which he indicates by the symbol “^X”, in order to say “the being of X”, is thus:

$$^Ua = ^Va$$

In other words, both have the same extension, they denote the same thing. In the article, Priest does not directly reformulate that with the T-schema, but we can take the risk of saying the following thing. Let Ua and Va be the names of *Ua* and *Va*. Moving to their *being* means affirming that:

$$T(\underline{Ua}) \Leftrightarrow T(\underline{Va})$$

57 Ibid., p. 407.

93 “To be and not to be – that is the answer”...

Following the equivalence

$$T(\underline{\alpha}) \Leftrightarrow \alpha$$

We can then affirm that

$$Ua \Leftrightarrow Va$$

Moving to “being”, of which he says, in the article, that it is money, that is in reality capital (including money when it functions as capital, that is to say according to the regime of self-valorization), is therefore moving to the truth of dialethia: the underlying unity of the difference between Ua and Va . An ideal example: the idea of *being* and the idea of truth are completely in parallel, which shows again that the question of truth is not a logical question: be it an ontological question as in Hegel, or a practical question as in Marx, non-dialectical logic is not, for Priest, the place of truth, but only a place of its manifestation.

In a posterior text, Priest responds to an objection⁵⁸ addressed to this double example. The objection consists of saying that the account of the simple form of value, the exchanged commodity a (20 yards of linen) and commodity b (a coat) with which it is exchanged are respectively the exchanger, exchange value, and exchangee, use value. The objection consists, simply, of saying that here there is no contradiction *stricto sensu*. Yet, this simple form of value is only a moment, the simplest abstraction (“the simplest, the most isolated, or the most accidental form” says Marx), of the exchange process: this moment never appears really alone. The real exchange of a and b is always symmetrical, both are exchanger and exchangee, and for this reason commodity is use value and exchange value. And it is not only the exchangee, but also the exchanger that is always both: use value as exchange value.

This is the reason why for Priest, the method of *Capital* concentrates the major stakes of all logical dialectics (quite independently of all formalisms). There is nothing original as such about this idea. What is interesting here is to see the way Priest articulates, on this point, the conceptual analysis and the goal of formalization.

These three examples (movement, alienated labor, commodity) are used by him as matrix of what he names dialetheias, i.e. logically true and untrivial contradictions, responsible for inconsistency, but expressive of a fertile paraconsistency. For him, the exact nature of dialectical

58 Priest 1990, p. 468-475, Marquit 1990, p. 147-166.

contradictions is given by the general form of “dialetheias”⁵⁹. Not only we have

$$(a = b) \wedge (a \neq b)$$

But in reality, we especially have

$$(a = a) \text{ c } (a \neq a)$$

Which means
Unity within difference.

This is the form of dialectical contradiction to which the others are boiled down (and on this point, Priest opts, naturally, for the thesis of the continuity between Hegel and Marx). The two main forms of this unity within difference are the followings:

- (1) The identity of one thing with its opposite-contrary: one thing is identical with itself in that it is different from itself.
- (2) The fact of one thing being F and $\sim F$ at the same time. Thus movement: the state of movement is one based on which a body which is in a certain place is no longer in this place: it is A and $\sim A$ simultaneously. Therefore, the fundamental signification of dialectical contradictions of Hegel and Marx is, for Priest, this logical signification, but a “logic” stuffed with the total weight of *one* world.

2.5. 3. Teleological Determination and Praxeology of the True

According to Priest, we can describe all states-processes of change starting with the form $\sim A = A$, seen under the intensional angle of the move to the opposite, from “going over”⁶⁰ of A in $\sim A$. For Priest⁶¹, all dialectical contradictions are therefore instants of the unity of opposites. The poles of dialectical contradiction have a stronger relationship that a pure and simple extensional conjunction, because a and b , even if different (thus, in this instance, of a and $\sim a$), remain identical. The dialectical identity is therefore an intensional identity: the relation that exists between the two poles of a dialectical contradiction is not static

59 Priest 1990, p. 410-412.

60 Ibid., p. 411.

61 Ibid., p. 412.

but dynamic: this question of process is essential. The classical concept of contradiction is dominated by the extensional vision: in $(A \wedge \sim A)$, there is no essential relation between the two joined terms. So that we can eliminate the conjunction and affirm one, A, independently of the other, $\sim A$ (for example, in the elimination of conjunction in natural deduction). Here, dialectical contradiction necessarily emerges from an intensional vision⁶²: it is the internal relationship between the joined terms which is not captured/capturable by an extensional conjunction. For him, an intensional approach goes, therefore, hand in hand with an ontological monism which alone makes possible a thinking of contradiction as unity within difference – and beyond the idealism/materialism opposition, the thesis of a radical monism, and therefore the thesis of reality as antagonistic totality, is shared by both Hegel and Marx.

To sum up, for Priest, the Tarskian convention-T does not offer a characterization, even only implicitly, of truth⁶³, even if we can consider it as offering the meaning of α and the implication for α to be true at the same time⁶⁴. It captures logical relationships between sentences, but these sentences emerge from a practice, and truth concerns the way these sentences are used, pronounced, within the framework of this practice: yet, truth is the telos of the fact of affirming (just as playing a game has a goal: winning): whence, the “teleological” determination of the true⁶⁵ with which he goes along, that is a conception of the true as being always situated, truth exists only for those who seek it, for those who turn it into their telos, and that is, necessarily, part of a practice. This amounts to proposing a true concept of truth. Whatever the precise conception of this semantics be, the idea is that the truth or the falsity of an affirmation stems from a relation to the *existence* of something which either is the case or not. But trying to know what is the case and what is not “deobjectivizes” the question, and situates it: without going into details, the foundation of the teleological semantics is therefore praxeological, practical. The unity of the real, of the discourse and of the practice summons a monist conception: the foundation of dialetheism is therefore a monist “metaphysics”, that is to say an above all Hegelian workmanship, and it is only in the midst of this unity-totality that the question of truth is posed in an always situated and oriented manner⁶⁶.

62 Ibid., p. 396.

63 Priest 2006a, p. 61.

64 Ibid., p. 60.

65 Ibid., p. 62, the formulas comes from Priest. Cf. Priest 2006b, p. 43-44 and p. 47-49.

66 This is the result to which whole of ch. IV of *In Contradiction* leads.

96 “To be and not to be – that is the answer”...

3. First Broadening of the Problem: the Monist Stake Behind the Epistemological Debate

3.1. Conjoined Results of Both Studies: Divergence of Orientation and Pseudo-Dialectical Convergence in Doz-Dubarle and Priest-Da Costa

The results of our study from 2010⁶⁷ were the following. In **Doz-Dubarle**, the operation consists of, for seizing the *Aufhebung*, instituting the term null Λ and the operators of “deposition” and “relevement”⁶⁸ as two operators of “negation” adding themselves to the negation understood or treated as the algebraic relation of complementation (for the new terms added up to those of the propositional calculus, which operate with the if-then connector, that is the traditional implication). In the categorical version: (1) If we have in mind the formalization project of Lautman sketched by F. Zalamea, **AND** the fact that Lautman does not consider that there are real contradictions, then in a certain way the problem disappears all by itself, given that his dialectics, rejecting internal negativity, is nothing but a “pseudo-dialectics”. On the other hand, (2) the willingness to formalize the unity of contradictions within category theory (Lawvere), if that should be in a really Hegelian sense, maintains the problem in all its acuity. Doz and Dubarle are, first of all, closer to Hegel from the point of view of speculative literality: they seek to come up with a formal model of the *Aufhebung* by giving a formal existence to the movement of the negative by which the abstract universal, by the mediation of its particularistic negation, is actualized in a negation of negation, in the concrete universal which is the singular. Simultaneously, they explicitly move away from the speculative spirit because their profession of faith is clearly logicist: for them, the standard condition of rationality is the possibility of translating in a formal-logical language, which leads them, against Hegel, to transform the dialectical negative in a manner that consists of making it disappear. The artificial character of their project shows that this magnificent construction has an “art for the sake of art” side to it, of which we could say that it attests to the simultaneously ethereal and indecisive character of their wish to move beyond the historical conflict of the two dialectical and analytical rationalities. This is revealed, moreover, by a certain primacy of syntax over semantics, that is to say, an indetermination at the semantic level that we indirectly established in the 2010 study.

67 Barot 2010

68 In French, “opérateur de deposition” and “opérateur de relèvement”.

97 “To be and not to be – that is the answer”...

As for Da Costa and Priest, they are at first less Hegelian because more distanced from his speculative literality: they do not seek to formalize the movement of the negative, but to show evidence of the fact that contradictions, as untrivial unity of contradictory statements, are, with certain conditions, logically thinkable. But simultaneously, their profession of faith is anti-logicist: their goal, in any case in Priest, is not so much to guarantee the rationality of Hegel by showing that he can be duly formalized, but to draw lessons from the fact that Hegel is right for their logico-mathematical domain, namely that there are, in reality, contradictions or related configurations, that we should accept this fact, live with these contradictions-paradoxes: giving them currency in mathematical logic is only driving in nail. This time, contrary to Doz-Dubarle, we see that paraconsistency, especially in Priest, expresses a strong empirico-metaphysical proximity with Hegel. (3) Because of that, they are more convincing than Doz-Dubarle in that they make their apparatuses work and produce results, unlike a quasi-aesthetical construction: paraconsistency produces knowledge. In other words, the formalization of dialectics is useful for science, whereas in Doz-Dubarle, it ratifies the scientific model against dialectics, by pretending to do this for the sake of dialectics. One of the points that highlight this big divergence is, on the one hand, the fact that tricky questions of logic and mathematics, like those of “their” philosophy, are approached and worked on head-on, and, on the other hand and correlatively, that there is a big work of semantics in paraconsistency, in the technical sense of plurivalence: {T, F, P}, in the metatechnical sense of a teleological conception of truth founded on a monist metaphysics, things that are merely sketched by Doz and Dubarle.

This does not prevent the Priestian operation to be, as much as that of Doz-Dubarle, the sign of a pseudo-dialectical victory. It makes the negativity disappear as well. From a Hegelian point of view, the unity of contradictions is a result posed from the movement of the negativity, and it is the dynamic work of this unity that leads it to the *Aufhebung*, their simultaneous preservation and abolition as such. It is because the negativity is internal to a determination that the latter can pass into its contradiction and unite itself with it. Yet, paraconsistency deals with, in Priest but also in Da Costa, the passage only conceptually and speculatively: the only thing with which they deal logically is the result. This amounts to hypostatizing, at the logical level, the result with regard to that from which it results: to stiffening it, there again, in exteriority, by axiomatically characterizing it, and by stipulating the analytical modes of its manipulation. Priest does try to smooth out the passage from the ontological to the syntactic via a semantic theorization riding two horses

at once. This does not prevent the new paraconsistent negation from being fixed, at the logical level, as an operator formally independent from that on which it bears, and this forbids us from thinking paraconsistent contradiction as an internal scission of a semantic unity: the intensional foundation of this unity is extra-logical. That is how the properly logical unity of contradictions remains the fruit of the combination of exterior elements.

To sum up, these two formalizations make the negative suffer the same treatment: to formalize it, it is necessary to abolish it as dynamism of interiority being at one with the process of actualization, and to fix it as an object or an operator formally independent of the “content” on which it bears: this contravenes, by principle, its speculative signification.

Here, formalizing the negative is making it disappear, by imposing a condition of manipulable exteriority on it, thereby destroying its interiority and its procedurality.

3.2. Returning to the Intertwinement of the Layers of the Problem

In a way, with regard to what we could keep in mind from Doz-Dubarle, the spirit of the formalization initiative seems to be reversed with Priest. Whereas in Doz-Dubarle the formalization of dialectics was supposed to attest to and extend its rationality, in Priest it is the reality of dialectics that attests and enjoins to its rationality by making it logically explicit. In other words, in Priest, the operation of the formalization of dialectics relies, finally, on that which, in Doz-Dubarle, it was supposed to guarantee the legitimacy.

This circularity, as temporalized discursive and historical process, refers, evidently, to a spiral shaped structure, and that, we have seen it, characterizes in particular that of the formalization of dialectics. In other words, the formalizations of dialectics are (1) an example of the general problem of formalization understood as a traditional knot in the logico-mathematical scientificity at large; (2) a particular historical illustration of a particular conflict of rationality which illustrates, itself, the traditionally clashing structuration, the traditionally paradoxical regime of scientific progress; (3) a way of formally characterizing the paradoxical regime of scientific progress⁶⁹. These levels are embedded and mingled around a same problem, the historicity of scientific like that of conceptual objectivity. The spirality, objectively highlighted on the occasion of or

69 And this rising reflexivity is again redoubled when paraconsistency is firmly used to formalize the regime of historicity appropriate to this intermingling. This is what is tried by Woods 2003: modeling the conflictual structure of the process of the progress of scientific discovery, understood as the strategy of identifying and resolving conflicts.

concerning dialectics, illustrates the dialectical and historical spirality of objectivity in general, and this dialectical historicity of objectivity is expressed and embodied in conflicts between theories, schools, and actors, that is to say *materially*.

We saw that it was necessary, for Priest, to invoke “semantic closure” so that he may speak not only of paradoxes but also of contradictions in the strict sense, i.e. intensional from the logical point of view. Yet, semantic closure belongs to natural languages. Priest insists on the naturality of semantic closure, taken then as the indication of the mundane reality of such a closure: there is nothing beyond language just as and because there is nothing beyond the world or history, space and time. Here, the founding monism is the thesis of the unity of the world, of history and of reason, of the real as a natural-historical process, and in it of thinking. And effectively: how can we say, fundamentally, that two things are contradictory if their relationship does not express their community? Only determinations having the same origin, membership or nature can be really contradictory. This is the Hegelian thesis: there is negativity because, contrary to Plato or Lautman, the Other emerges from the Same. And yet, the cosmological unity in Hegel is the unity of the Concept, that of the self-realizing universal, of the infinity working in the midst of the finite: such is the foundation of idealism. Marx deconstructed this idealism by showing the unity of history and nature, with the human history as its real natural history, as unity and totality in becoming translating itself by class struggle, i.e. the work of the contradiction between work and capital, in other words between work and itself via the mediation of the social and natural world: thus the two examples employed by Priest, after the one of movement.

But if Marx challenges the idealist foundation of monism in Hegel, he does not found this monism: he does not really establish, does not demonstrate, as materialist, this monism conditioning the possibility of the existence of real contradictions. This is the meaning of the main critique addressed by Sartre to Marx and to Marxism in his *Critique of Dialectical Reason*: if history is a totalization, i.e. one (even if its meaning is out of reach), then Marx is right. In other words, Marx is practically right, but that which theoretically founds this practical reason is not explicit in Marx: thus, as materialist, it is necessary to reactivate it by qualifying once again the Hegelian operation of founding the unity of the real. It is therefore necessary to establish under what conditions real contradictions can exist: Sartre tells us that the major corollary of the concept of contradiction is that of totalization. Establishing that history is one and one totalization, it is giving oneself the means to demonstrate that contradictions are not ways of talking, but forms and structures

immanent to society and history. This is the goal of the second volume of *Critique of Dialectical Reason*.

Now it is evident that this necessity of refounding the unity is itself a historical necessity. It is therefore necessary, now, to articulate what is at stake in monism and historicity by showing that they are one. But it will be especially interesting to show this at the very heart of the determination of the sense of the problem of the formalization of dialectics, and not as a mere extension to which the latter would lead, as a beyond of itself.

4. Outline of the Politico-historical Sense of the Formalizations of Dialectics. Second Broadening of the Problem

Given that, naturally, what I have done so far has only dealt with two types of formalizations of dialectics, I could draw from it not real, but particular conclusions. This is why it is necessary that we pursue the examination based on other examples of formalizations. But as it is, not as proofs but as suggestions in order to contribute to the debate, I will allow myself to broaden and to put into perspective, in the following remarks and by means of an outline of a politico-historical interpretation of the affair, the pseudo-dialecticity verdict put forward above. At the risk of being repetitive, but in order to limit as much as possible the risk of missing indispensable mediations, I shall begin by clearly recapturing the way I have tried to construct the problem.

4. 1. From What is at Stake in Formalization to the Specific Issues of the Formalization of Dialectics

The logico-mathematical production *stricto sensu* has been required, since more than a century, to be regimented in formal systems which have become, if not the whole of objectivity, at least the guarantee of its logico-mathematical character, i.e. quite simply of its scientificity. That has not always been the case in history, but the idea of reorganizing on a purified, as unequivocal and systematic as possible, basis, by short-circuiting the epistemological obstacles of intuitive or empirical kinds, is not new: from that point of view, independently of the difference of nature between the Euclidian axiomatic and the Hilbertian axiomatic, the objective, the general purpose, and the passage to formalization in the contemporary sense, the specific, that is historically situated, form that the pursuit of this objective has taken is comparable. Whence the question: under what conditions, on what occasions, and based on which

purposes, and what, do we formalize? All formalizations are committed to giving exactitude to that which is not or is not sufficiently exact, and tend to reduce certain previously experimented conceptual problems to technical and calculative questions. But if we keep in mind Gödel's 1931 result, all new formalizations conjure up, in their own midst, new problems that they are not apt to settle, i.e. through which they stumble over their relative and limited character, that is, over their own limits; and these limits are always, in one way or another, the expression of the existence of a conceptual or speculative residue which is irreducible to mere technicality.

The movement of formalization as such is precisely that by which the existence of a stratum simultaneously constitutive of the objectivity of mathematical and irreducible to technico-formal procedures is certified. That is why the multiple adventures of the Gödelian result, from Lautman to paraconsistency, invite us to reread, as a tool, the previous history of the logico-mathematical theories as the conjoint history of attempts at instituting the epistemological rupture, and the impossibility, probably irreducible, of successfully completing these attempts.

The object of the logico-mathematical praxis is therefore structurally unclear: that of which it is responsible goes beyond it, its "object" is not reducible to "its objects". It is this excess of the object that, in so far as it attests to the irreducibility of the speculative to the technical and forbids all forms of positivism (and positivism is even more miserable when it is implicit), blurs, and even abolishes, at one moment or another, the frontiers between the philosophical and the mathematical. Since Plato, "dialectics" at the same time says and baptizes this excess of mathematics over itself, in its own midst, this beyond of itself, and tries to characterize the conditions and the rational place of this diction. From this point of view, dialectics is (1) the major philosophical discourse, in history, of the impossible self-foundation, and correlatively of the impossible self-formalization of the logico-mathematical, and (2) the kernel, starting with Hegel, of the discourse of the historicity of the latter's objectivity, i.e. of the self-corrective and continued dynamic of the combinatorial of its different kinds of foundations.

When this dialectics itself starts to be submitted to formalizing attempts, the problem is enriched with an additional level and signification, which accentuates the *historical* character of the problem of mathematical objectivity. If, on the one hand, dialectics is the discourse of the irreducibility of the enigmas of mathematics to their technical theorization, and if, on the other hand, the problem of formalization is traditionally a prism, a privileged occasion, for examining the stratification of mathematical objectivity, it is only with the attempts at

formalizing dialectics that these two adventures become one and the same adventure: these attempts come to conjoin and telescope, thirdly, the first two problems, and thereby produce a new, which reveals the rising reflexivity, representative of a certain historical stage, of the first two problems.

4.2. Elements for a Radical Historicization

The initiative is philosophically problematical, at the very least paradoxical, and as technically passionating as defective from a Hegelo-Marxist point of view. But it is necessary to understand its motivations, a concern for legitimization on behalf of the dialecticians, an unrelenting willingness for capturing that which eludes on behalf of the mathematicians, but also to understand that which renders possible these motivations. Evidently, this concern and these conditions are historically contextualized. Not only the problem of formalization is never posed outside of the particular historical experience of a problem that resists and insists, and arouses its deployment, but here, in the case of dialectics, we are faced with an entirely new, symptomatic and historical, paradox.

4.2. 1. Two Historical Conditions of the Problem

Imagining the manifestation of this formalizing will was not possible before

(1) The duality analytical reason / dialectical reason losing its complementarity face (Plato) to assume, with Hegel, that of rivalry. As the first necessary condition of emergence of the problem, the duality of rationality should take a conflictual form, and this conflictuality, magisterially elevated to the concept, and kept, simultaneously, in suspense in the speculative order by Hegel, is that of the booming capitalism of the 18th century, worked by an explosive antagonism.

(1) But there is also a second necessary condition. It was necessary, moreover, that this rivalry, understood as mutual condemnation of irrationality (dialectical reason because it accepts real and discursive contradictions as fertile principles, analytical reason because it mutilates the complexity of the real), be considered, at least in the spirit, as soluble without reductionism.

In other words, it was necessary (i) that the criteria of legitimization provided by the logico-mathematical manage to find, under certain

conditions, grace in the dialecticians' eyes, and (ii) that the criteria of legitimization promoted by the dialectical idiom manage, under certain conditions, to find grace in the logicians' eyes. Now, this second double condition is proper to the 20th century. Sub-condition (i) has been satisfied by a growing concern and awareness, among the scientific workers and/or their epistemologists, of the historicity of objective knowledge, of its nonlinear and non-mechanical, but critique and "recurrent" temporality: the crisis in the foundations of the first half of the 20th century, the contradictions of set theory, the limitations brought about by the incompleteness, but also the paradoxes of space and time brought to light by wave mechanics, then quantum mechanics and general relativity, have promoted, in spite of its variegated and complex nature, the acceptance in the scientific and epistemological field of the idea based on which reality is made, if not of contradictions, at least of paradoxes, of tensions, to which the dialectical idiom is certainly adapted. The pregnancy of the dialectical and historicist banner in the post-neo-Kantian French School, and a little bit later the historicist currents, even if in minority, of the Anglo-Saxon field itself, like the one represented by Kuhn, bears witness to this process.

With regard to sub-condition (ii), it was necessary that the will to formalization in the strict sense be completely integrated, even among the dialecticians, as constitutive, if not exclusively, of scientificity: the fact that a certain opening, a certain flexibility, has been attached to the logico-mathematical model of scientificity (totally absent, besides, from logical neopositivism, which is why it fought against it such forcefully), is that which has rendered this revival of legitimacy possible in their eyes⁷⁰. This is why the formalizations of dialectics could not be born in any moment other than after the Second World War, during the second half of the 20th century.

4.2. 2. Diagnostic Politicization

My working hypothesis, that here I will only briefly sketch, is that these attempts, beyond this specific contextuality, have constituted a local and ethereal path to get out of the immobilism of a cold war between analytical and dialectical reasons, of the ossification of an antagonism between "a western rationality" and a "dialectical materialism", given that both are characterized by their historical failure at constituting themselves as the whole of a rationality of thought and of the society.

70 This is something that Marcuse does not take into account, probably out of ignorance, but, in his defense, because the ideology that he criticizes is absolutely disconnected from these marginal subversions of the (neo)positivist model.

These formalizations show and give evidence to a certain state of culture and thought which has tried to unite, once again, but prudently, what had appeared to it as two excessively and damagingly entrenched camps. But if they are the expressions of a paradoxical historical moment from the point of view of their immediate theoretical significations, they stand out, on the contrary, as the non-paradoxical representative expressions of a social, intellectual and political need, induced by a certain state of history: getting out of the cold war by reconciling the opposites. On the one hand, if we leave aside their technical and conceptual results, we can defend, in them, the willingness to reunify the rationality without unduly homogenizing it. But on the other hand, once we take their results into account, we cannot but call into question their pretension, i.e. their way of envisaging this reunification: the dissolution of the negative by its institutionalization that they effectuate likens them to an indirect form of social-democratization of the problem of communism and of the revolution.

If the goal, as we have seen it, is not to evade "inconsistency" but to channel it by localizing it for avoiding that it affect the whole system, are we not dealing with an initiative of hijacking? Does it not amount to accepting the contradiction precisely to remove its explosive character?

This bundle of initiatives is therefore a certain face of the form of capitalism's objectivity during the second half of the 20th century: similar to the bourgeois arts of the "affirmative culture" the way Marcuse understands them, they constitute theoretical forms in which the contradictions of the society have partially found a way of expressing themselves. But if terms such as "non-standard" or "non-classical" do correspond to these initiatives, by meaning heterodox, heterodox is far away from the signifier oppositional. It will be necessary therefore to dig, in the future in particular, this hypothesis: that the axiomatic-set theoretical paradigm of Bourbaki has constituted the principal, dominant, orthodox and recalcitrant from of objectivity, as much of paradoxes as of contradictions, of the statist imperialist capitalism of the 20th century⁷¹. I would suggest that the formalizations of dialectics constitute an "alternative" micro milieu to this dominant paradigm, but working on the same ground, i.e. with its main pre-requirements, and leading to the results that it expects.

71 It is not surprising, retrospectively, that Jean Dieudonné, the "philosopher" of Bourbaki, *de facto* leaves meticulously out, in his reiterated homage to Lautman, precisely all that was related to dialectics in the latter.

4.2. 3. Prospective Politicization: the Unlocalizable Character of Communism and Formal Destinies of the Negative

Let us conclude with certain prospective remarks. On the one hand, we could generalize the topic by saying that the negative is consubstantially stubborn towards formalization, that all projects of institutionalization come up, by definition, against failures, and that therefore all projects of this kind are intrinsically doubtful and liberticidal. Sartre or Marcuse would possibly go in this direction. But if we can agree on the foundation of such an impossibility on the side of the negative, that should not make us be biased against the inventiveness to come of the human species. Imagining the possibility of a formalization of dialectics capable of satisfying the negative as negative is not more absurd than imagining an institutionalization of communism as the realized association of free men having abolished social classes, without presupposing, at any level whatsoever, a teleological and linear history. If we maintain the purely analogical character of both problems, we can effectively think that the soluble or insoluble character of one of the two problems does not harm, by any means, the soluble or insoluble character of the other. If we affirm, on the other hand, that the “epistemological” problem is only a particular expression of the politico-historical problem, then we are dealing with a unique problem only. The possible Sartre-Marcusian sanction leads to a difficulty: the resolution of the problem tends to become an irreducible point of flight, a pure and simple *regulative* idea.

Do we really have any proof to support the affirmation based on which all institutionalizations carry in themselves a tendency towards ossification, towards inertia, towards the repression of the negative? Or does it remain possible to imagine forms of institutionalization of the negative which would have the virtues of stability and rational regulation without confining it to a sandbox? And, then, which tools and practices can constitute the kernel of the revolutionary transition which would lead, as Marx, Engels, Lenin and their consorts used to hope, to such an institution of freedom? The “the dictatorship of the proletariat” as a residual state, emerging from the destruction of the bourgeois state, working towards its own withering away, as state and anti-state at the same time, in short, as a transitorily contradictory institution, is it condemned to failure? The preconceived view of revolutionary materialism consists of saying that the dialectics of the praxis of humans to come is not written in advance, in its successes like in its failures, that a test, an ordeal, is not a proof⁷².

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72 Cf. Barot 2011

Two conclusions therefore. First of all prospectively: nothing prevents, in principle, the realization of a formalization of dialectics. But the good form of the negative, just like the real organization of authentic communism, are yet to be found. Then “diagnostically”: the cold war is behind us: the USSR no longer exists, its *diamat* neither. But that does not mean that the stage of society has changed, quite the opposite: capitalism is still there, more than ever. The new attempts at formalizing dialectics are affected by a profound ambiguity: they keep on trying to legitimize the revolutionary principle with the means of the dominant rationalism, but this remains a fundamentally conservative goal, because it aims at channeling and institutionally regulating the aforementioned principle which consists of nothing more or less than abolishing it. Today, this ambiguity remains the element restricting what is most essential in the left-wing practices and thoughts. It is only by pursuing the diagnostic examination of their tensions, and by concretely working, by contrast and directly, towards such prospective aims, that the former can be surpassed, vigorously⁷³, by the latter at all levels, including at those which are apparently most immunized against all politicization, and which are dealt with by the formal sciences. What is at stake in this affair, *in fine*, is to remember very well the reason why the essence of dialectics, in its materiality and its history, as emphasized by Marx in his 1873 postface to the republication of the first volume of *Capital*, is “critical and revolutionary”.

Translated by: Sina Badiei

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73 I have already invoked, in the study from 2010, the necessity of examining in detail Alain Badiou's proceeding in *Logics of Worlds* from 2006, which contains nothing less than a formalization of “dialectics” by means of a particular segment of category theory, and of a revisited conception of dialectics. I can only reiterate, for want of anything better for now, this necessity, just as it would be necessary to deal with the synthesis and the proposed readings of the problem by Marconi 1979, as well as to deal with the works of the Polish and Russian schools, in particular the works of Ilyenkov 2008.

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