Notes on the Equivalence between Ontology and Mathematics

Burhanuddin Baki

Abstract: This essay collects some thoughts on Alain Badiou's thesis 'ontology = mathematics' and his mathematical metaontology. Issues such as Badiou's selection of mathematics are discussed and evaluated, as well as the possibility of extending the philosophical project towards other mathematical fields. We analyse the metaphysics, or lack thereof, given by this metaontology. We also provide some points of comparison with the analytic philosophy of ontology.

Keywords: mathematics, ontology, Alain Badiou, set theory, metamathematics, metaphilosophy, *Being and Event*

1. The following sequence of schematic remarks[1] provide a précis to some of my recent meditations and assessments regarding Alain Badiou's 'ontology = mathematics' thesis[2] as well as the mathematical metaontology that arises thereof. The relevance of the thesis to the issue at hand, the philosophy-science relation, is obvious when we realize the indispensable roles played by ontology for philosophy and by mathematics for science.

2. The core consideration of philosophy, if we accept Heidegger's intervention[3], is the question of Being qua Being. Ontology is the name of the discourse that focuses on this question. Badiou's thesis audaciously posits that this discourse is what, all this while, we have been calling mathematics. To lay claim to mathematics is to lay claim to a history, archive and ongoing research enterprise that includes arithmetic, geometry, calculus, algebra, probability theory, combinatorics, statistics, topology, set theory, and so on. It also includes the methodological tools shared by fields in applied mathematics, as well as the physical and social sciences.

3. Equated with mathematics, ontology is no longer a subfield within philosophy, even though Being qua Being still constitutes the core question of the latter. Philosophy can only concern itself in a roundabout way, on at most a second-order level, with the essential question of Being qua Being by pursuing mathematical

1 Research for this work was supported by the Universiti Sains Malaysia Short Term Grant [Reference number: 304 / PHUMANITI / 6313326].

2 Badiou 2007a.

3 Heidegger 1996.

truths that erupt following from unexpected and high-impact events. Such truths take the ontological form of generic sets whose infinite weaving is a truth-procedure by a subject. In addition to mathematics, philosophy compossibilizes truths from various other domains, which include art, politics, love, and the other sciences.

4. Badiou's equation must be distinguished from two others given by him in *Being and Event*: 'Being = multiplicity' and 'ontology = ZFC'. Note that the three equations are identities, not predications. Each left side is proposed to be exactly identical to its corresponding right side. Every ontology is mathematical, and every mathematics is ontological, without any excess, exception or counter-example on either side. The Beingness of every being is its multiplicity and the multipleness of every multiple is its Being. And so on for the third equation: ontology is precisely ZFC and vice versa.

5. The second equation, 'Being = multiplicity', arises out of two observations. First: every entity is a multiple, a collection of elements. Second: every entity always exists situationally with respect to another multiple. To be in a room is to be an element among the multiple of elements in that room. Pegasus exists in the collection of entities inhabiting the world of Greek mythology. The number 32 exists in the Peano situation of arithmetic involving whole numbers. To be is to be a multiple and to belong to another multiple. The second equation takes these observations further by daringly postulating Being qua Being to be *essentially* multiple. All there is to the question of ontology is the question of multiplicity.

6. Mathematics has its own name for the multiple: the set. There is a branch of mathematics devoted towards studying sets: set theory. The natural corollary to the equivalence of Being with multiplicity is the reduction of ontology to set theory. ZFC, the collection of Zermelo-Fraenkel axioms plus Choice, is a formal system for that theory. Hence, the third equation 'ontology = ZFC'.

7. All of this does not mean that Being is the same as set or that concrete beings are formed by mathematical objectivities. Being qua Being is linked to the side of inconsistent multiplicity, the count-as-one operation which is prior to the consistent multiplicity of concrete beings.

8. A pure set is a multiple containing other pure sets, all the way

CRISIS & CRITIQUE / Volume 5 / Issue 1

down to the empty set that contains nothing. In principle every mathematical entity can be constructed as a pure set and the entire edifice of mathematics can be reduced to operations involving pure sets. Ontology is, at the most minimal level, the theory of the pure multiple, of multiplicity itself as such. Set theory is a meta-mathematics, a mathematics of mathematicity.

9. Much can be gained for our understanding of ontology by analysing the ten ZFC axioms and their implications. We can examine the universe of Being qua Being by examining any universe of sets where ZFC holds, with the most minimal being the universe of pure sets. Since the fundamental basis of philosophy can only proceed metaontologically via a program of compossibilization of ontological truths, and since ontology reduces to ZFC, therefore any philosophy must proceed via a close examination of all the definitions, theorems and proofs that the existing mathematical literature has provided about those axioms, as well as examination of the various models that satisfy ZFC. Badiou's accomplishment in *Being and Event* is precisely this.

The Usual Quibbles

10. As we go through the three equations, 'ontology = mathematics', 'Being = multiplicity' and 'ontology = ZFC', we see that that later equations are specific articulations of the consequences to the earlier ones when transplanted into particular mathematical domains. The second equation proceeds from the first when we ask for a meta-mathematics and get set theory as a mathematics of multiplicity. The third equation proceeds from the first two when we ask for a formal axiomatic system for that meta-mathematics and get ZFC.

11. Mathematicians with background expertise on various meta-mathematical systems might find quibbling with the selection of ZFC to be hard to resist. Why set theory instead of type theory, category theory, homotopy type theory and so on, with each theory offering alternative ontological units for the multiple? Even if we use set theory, then why the ZFC axiomatic system instead of Kripke-Platek, Morse-Kelley, Quine's New Foundations, Tarski-Grothendieck, and so on? And why limit our expressive language just to first-order logic?

12. Say we opt for meta-mathematical theory T. We believe it to be better than ZFC for axiomatizing not just mathematics but also ontology. Our belief was established through an extensive and careful analysis justifying the benefits and strengths of T. Nevertheless, as far as philosophy and metaontology is concerned, we have only begun. The task now is to study the mathematical truths of T and compossibilize them, along with truths from other domains, into a rigorous, robust and compelling philosophical system. This is the creative task of philosophy. Understanding the mathematics is not enough. What must be done later is to construct a general philosophy from the mathematical truths. And to do so without the product being simply a philosophy of and about mathematics.

13. The peculiar characteristic of philosophy under Badiou's methodological schema is that it can only parasitize on the truths erupting out of other non-philosophical domains. Philosophical activity is always essentially trans-disciplinary, but without being subservient or sutured to other fields. It originates from indispensable but novel encounters with external domains. For example, Jacques Derrida's famous essay 'Structure, Sign, and Play in the Discourse of the Human Sciences'[4] might be structured as a reading of Claude Lévi-Strauss's ethnological studies. But it is also a creative philosophical work that stands on its own, a work that inaugurated the influential philosophical orientation of what we still stubbornly call 'post-structuralism'.
14. Some philosophers, like Descartes, Leibniz and Russell, were great mathematicians. They were also often directly responsible for the invention of the mathematics that later conditioned their respective philosophies – for Descartes, the cogito; for Leibniz, the monad; and for Russell the original philosophical paradigm for analytic philosophy. Some, like Deleuze, Spinoza, and Plato, were philosophers who were also great scholars of mathematics. Their philosophies might have been conditioned by mathematical results that predate them by several decades, even centuries. The greatness of their thought follows from the intrinsic quality, forcefulness and innovation given by their ideas. Philosophy has its own disciplinary sovereignty that stands apart from other domains. If someone can devise a new philosophical system conditioned by Voevodsky's meta-mathematics of univalent foundations[5], then all the better.

4 Derrida 1993.

5 Voevodsky 2013.

CRISIS & CRITIQUE / Volume 5 / Issue 1

Ontology without Metaphysics

15. The 'ontology = mathematics' equation is an audacious thesis. Perhaps almost as audacious as the most controversial philosophical equation of all, Spinoza's 'God = Nature'. Two discourses, erstwhile believed to be essentially separate and occupying different disciplinary regimes, are suddenly postulated to be equivalent. What is more, the main question for philosophical foundations 'What are mathematical objects?' becomes not only solved but dissolved.

16. We now ask this: what is stopping us from making our own maverick move and thereby positing 'metaphysics = mathematics'?

17. The two discourses – ontology and metaphysics – are sometimes invoked and used interchangeably. Some philosophers take them to be entirely separate. Some see their domains overlapping, or one being a subfield of another. At any rate, metaphysics investigates questions that aim to compose a complete understanding of fundamental reality. These questions may or may not include the question of Being qua Being. Metaphysicians study more than that by tackling not only questions regarding what things are, but also how they work and how they interact amongst themselves on a fundamental level. Can mathematics and a mathematical metaontology entirely handle these types of questions too?

18. In its radical reconceptualization as mathematical thinking, ontology divorces itself from some chief metaphysical concerns, particularly when they involve the issue of identifying some fundamental origin or some essential oneness. Since Being is essentially multiple and the one is not, then there is no ultimate ground, no fundamental reality behind Being qua Being. Laicized of any fundamental *theos*, the Great Outdoors do not appear to ontology like some bequeathment from *le dehors*. At least this is the case as far as the discourses of ontology and any metaontology is concerned, although this might not hold for philosophies conditioned from other domains of truth. Still, this refusal of the One remains when all these conditions are compossibilized together with a mathematical metaontology consistent with Badiou's equation.

19. Laicized from any metaphysics of lost origins, Being and multiplicity are empty signifiers in this mathematical ontology. Nothing is

behind or beneath them, for the count-as-one is the void. As a discourse, mathematics is meaningless. Lacking a power set (the ontological structure of what Badiou calls its 'state') the situation of mathematics lacks a proper semantics, an essential interpretation of its main vocabulary, particularly regarding the meaning of Being. Bertrand Russell defined mathematics as 'the subject in which we never know what we are talking about'[6]. It is the sole discourse that works without us knowing what we are referring or talking about.

20. This feature allows mathematics to escape the Heideggerian dilemma of the metaphysician's forgetfulness of Being. By investigating Being in an indirect manner without unifying or interpreting it, mathematics can avoid mere ontic thought. This is due to the splits between the axioms and its semantics, as well as certain features of mathematical axiomatics following from Gödel's Two Incompleteness Theorems[7]. The consequence of Badiou's equation is, perhaps, an unexpected accomplishment of Heidegger's dream for the deconstruction of metaphysics[8]. Mathematics is a discourse of Being without focusing on it directly. Moreover, in its refusal of unity, the philosophical system of mathematical metaontology accomplishes an immanent truth that, for him, is even more radical than what had been attempted earlier by Deleuze[9].

21. Insofar as it diagonalizes through the classical dichotomy between the mind and the Great Outdoors, the Speculative Realist issue of correllationism-vs-anticorrellationism is not relevant for this mathematical ontology and metaontology. If some external truth of fundamental reality ever announces itself and impinges on ontological thinking, its emergence takes the form of an event that is both immanent yet novel at the same time with respect to ontology's internal situation. Badiou provides a technical elucidation of this emergent process in his metaontological analysis of Cohen's forcing and generic filters[10]. The generic structure of truth consists of a novel and infinite multiple of existing elements.

6 Russell 2013, p. 75.

7 Gödel 1931.

8 Heidegger 1978.

9 Badiou 2000.

10 Cohen 2008.

22. Mathematics can only be equated with metaphysics if the attempt at a fundamental ground is removed. However the categorial architectures of existence provided by many meta-mathematics do provide some answers to many the usual metaphysical issues. Even though everything is basically a multiple, there are differences. We have sets, relations, functions, equations, geometric manifolds, graphs, formal languages and so on. Moreover, according to Badiou, a different meta-mathematics, topos theory, supplements the set-theoretic viewpoint and provides a way to understand the vertical relationships between multiplicities[11]. Other meta-mathematical foundations, like homotopy type theory or simple type theory could accomplish this as well, provided we do the work. The technical grunt-work of building, understanding and interrelating these multiples has already been done by the mathematicians, not the philosophers, and without some direct attempt as comprehending Being. Quite a lot of the 'metaphysical ground-work' has already been earlier delegated to the mathematicians. Philosophy parasitizes on the technical grunt-work of the mathematicians, but then seeks to do more with it within the domain proper to creative philosophical compossibilization.

23. It would an interesting project, which I will not pursue here, to choose the top ten most important classical questions in metaphysics and, provided they are truly questions for mathematical and philosophical thinking, examine whether they can be resolved using a chosen meta-mathematical ontology, be it set theory, category theory of so on. For anything left, we can then examine whether they can be tackled by a different domain for truth-conditions, such as from the other sciences, or by art, love or politics.

Occasionalism without God

24. As a theory of the multiple itself as such, ontology is reduced to the question of presentation. Or, to be precise, the *facticity* of being present or being absent. Multiples and situations define themselves solely by their count-as-one, by what is present or absent in their belonging-relation. The question of Being is sutured to the question of existence. The question is not *what* but *that* something is or is not.

11 Badiou 2009.

25. In this flat plane of Being, what we observe is a schema curiously akin to what might be called an Occasionalism without God. In the medieval perspective of Occasionalism, no entity is efficiently caused by another as each thing directly appears due to divine power. It is God who causes the second billiard ball to move, not the first billiard ball hitting it. In the ontology of multiplicity, this God is deemed to be either missing due to a doctrine of atheism or irrelevant due to the politics of laicization. Entities just exist, without any vertical relations of causality or supervenience with respect to one another, for a relation is just another entity, another presented multiple. Of the void of Being that is the anonymous count-as-one operation, nothing can be said other than its inconsistency.

26. We can accept this contingent facticity of Being as it is and leave it at that, without any further questioning or analysis. Or we can make this irreducible facticity as the unconscious Unsaid of this mathematical ontology and its corresponding metaontology. Here the impasse of the Real is not an entity's material quiddity, the deep night of its material soul, but the facticity of its existence, without any recourse to some analytic of intrinsic or essential otherness.

27. Pre-Badiou, the mysterious alterity of Being referred, first, to a God and then, later, to some secularized *autrui* (which is really the remnants of some stubborn religious or quasi-theological trace). Post-Badiou, the mysterious alterity shifts to the radical contingency of Being. We have abandoned the theological and onto-theological question 'What is behind it all?' for 'Why are things the way they are?'. Hence: in many post-Badiouian philosophies, some by thinkers grouped today under the Speculative Realist movement[12] and its offshoots, a renewed emphasis on the absolute otherness of this ontological contingency.

Comparing with Analytic Philosophies of Ontology

28. How does this mathematical metaontology differ from analytic philosophy? Does not the latter also have deep respect for mathematics and maintains the figure of mathematical rationality as a model for the clarification and structuring of argument?

12 Meillassoux 2010.

CRISIS & CRITIQUE / Volume 5 / Issue 1

29. As far as I know, under any methodological attitude that can be called 'analytic' (although this term does not name any centralized monolithic tendency) the philosophical treatment of Being qua Being never goes as far as to equate mathematics with ontology itself. There, the role of mathematics is only methodological and paradigmatic. This is despite – and perhaps also because – of the history behind the early roots of the analytic movement. Following from the developments by philosopher-mathematicians such as Frege, Russell, Moore, Wittgenstein and the Logical Positivists, early analytic philosophy is the consequence of a specific philosophical compossibilization out of the fields of mathematical logic and meta-mathematical foundations. We can easily see, for example, Russell's later philosophical work as a natural progression from his *Principia Mathematica* days.

30. The language and methodology of mathematical logic and meta-mathematics thoroughly permeates analytic philosophical thought, so much so that it could be argued that, in that situation, philosophy has been sutured to its conditions. I dare the risk of going further and posit that, for a large portion of the analytic school, 'first philosophy' is not metaphysics as Aristotle defined it, nor ethics as Levinas posited, but mathematical logic.

31. In Badiou's conception of mathematical metaontology following from his equation, ontological considerations can never be tackled directly, be it by mathematics or philosophy. The Heideggerian dilemma of Being means that ontological thought can never be realised head-on. Unlike in analytic philosophy, the philosophical questioning of ontology cannot precede formal mathematical concerns; one cannot simply construct a new philosophical thought about Being and then formulate it via some mathematical formalism.

32. Equated to mathematics, ontology exists as a sovereign discourse on its own, parallel but entirely separate from philosophy, who often drops by for a visit, like a journalist interviewing the aristocratic socialite for the latest news and gossip. The mathematician William Timothy Gowers writes that mathematicians, when they are doing mathematics, have no essential need for philosophy[13]. Like a hyena, philosophy can only, at most, come later and parasitize on mathematical truths, and only ones that originally erupt from the

13 Gowers 2006.

commitment to some event, but without producing any ontological theorem on its own. Philosophy can only be at most metaontological.

33. Let me attempt to develop this a little bit more with a brief comparison with the most famous essay in the analytic philosophical treatment of ontology, W.V. Quine's 'On What There Is'[14]. The first main point by Quine is that statements about the existence or non-existence of X are clarified using the rule 'To be is to be the value of a variable'. With the help of Russell's theory of descriptions, Quine applies the regimented language of what later became the mathematical syntax of quantified first-order formal logic to understand what we mean on the level of the statement that x exists. Being is discursively captured by the act of existential quantification ranging over some domain of discourse, some ontological commitment about what exists.

34. Badiou's concept of the situation-multiple can be compared to Quine's concept of the existential domain of quantification. They also both share this application of first-order logic. Whereas Quine is only concerned with meta-ontology on the analytic level of existential statements, Badiou wishes to understand the deep structure of Being itself as such. For Quine, mathematics is just a cognitive and rhetorical device for philosophical formulation and the clarification of statements. For Badiou, mathematics is precisely ontological discourse because Being and multiplicity are the same. Ontology is originally accomplished as mathematics by and for mathematicians.

35. Quine's second main point: ontological commitments are constructed based on the best results of the day from the natural sciences. In existential statements, the bounded variables range over a domain of discourse that is determined not from fundamental metaphysical inquiries based on first principles. An ontological commitment is constructed once we have determined some overall conceptual schema for accommodating all and only entities that are indispensable to the best scientific theories. The ultimate arbiters on existence are the natural sciences. This is not inconsistent with what Badiou has given us. The task of ontology and metaontology ends for the question of determining what is presented in the contingent physical world. Mathematics can only provide the overall skeleton for

14 Quine 1948.

the structure of Being qua Being. To use the semantics of possible-worlds, mathematics can only say what all possible worlds have in common, not what exists contingently in each. There is no overall interpretation for multiplicity in mathematics as there is no 'state' for the ZFC axioms. On its own, the most that ontology says is that there is the void.

36. Ontology cannot say whether an apple is made fundamentally of atoms or strings. But it can say, following from set theory, that Being is captured by the notion of set. A philosophical metaontology can be constructed out of the truth of that notion, which was Badiou's project in the first half of *Being and Event*. Moreover, Badiou's metaontology of Cohen's set-theoretic forcing provides a philosophical way for understanding the development of these ontological commitments as new scientific discoveries are made in fundamental physics. Of course, if categories, types or homotopy types are chosen instead of sets then that metaontology would be different. For Quine, the question of what is must be based on the best contemporary science. And perhaps for Badiou, the question of what-is-insofar-as-it-is must be based on the best contemporary mathematics.

37. If mathematics solely accomplishes ontological thinking, with philosophy only feeding on its carcass later, then what to make of other philosophical thinking of ontology by other non-mathematical thinkers? If we are unable to completely reject what, for example, Hegel, Kierkegaard, Heidegger, Sartre or Buddhist philosophy writes about ontology, then how to reconstitute its mathematical trace? Should the task be to dig for them in the archive, through careful explication? From the deep archival recesses of what mathematical literature did Heidegger produce *Being and Time*? Can we not reconstitute his ontology as metaontology and, better, find some unexpected mathematical theorem hidden within its textual unconscious? Could this be new and innovative mathematics, even by the high standards of contemporary mathematicians of today? Or would such philosophies be the un-mathematizable itself and automatically rejected as a thinking of Being?

Philosophy without Meta-Mathematics

38. Let me shift to a different register and ask: how to extend this

methodology of philosophical compossibilization towards results from other mathematical fields such as extremal graph theory, stochastic partial differential equations, or algebraic K-theory?

39. I am still unsure how to approach this task. Note that Badiou himself has only focused so far on a specific type of mathematics, namely meta-mathematical mathematics. To be specific, he examines the meta-mathematics of set, model[15], category, and topos theory, in addition to the meta-mathematics of numerality via a short excursion into the theory of surreal numbers[16]. The issue is whether a properly Badiouian metaontology, one that is consistent with his 'ontology = mathematics' equation, can also be constructed from a mathematical field that is non-meta-mathematical.

40. I suspect that it might be slightly easier, for various technical reason, to build philosophical systems out of properly meta-mathematics fields compared to, say, enumerative combinatorics, ergodic theory, or the study of elliptic curves. By seeking to provide a summation of mathematics via mathematical means, by asking mathematics to foundationally account for itself via its immanent contemporary tools, concepts and methodologies, the subfields within meta-mathematics are already philosophical both in their content and in the required skill-set for their comprehension. Moreover, by being a mathematics of mathematicity, fields such as set and category theory enable an immanent self-referential focus that enables a purer and more impeded access toward a philosophical thought of intrinsic Being. Another crucial feature for meta-mathematics is that they automatically allow for the breadth and universality of analysis demanded out of most ontological and philosophical meditations. Since all mathematical entities are in principle constructible as sets, then any theoretical analysis into the notions of sethood immediately involves the whole generality of mathematics.

41. (This lack of ontological generality is, in my opinion, one of the weaknesses of Deleuze's mathematical metaphysics, which is conditioned partially by truths from the specific fields of differential calculus and differential geometry[17]. Not all entities, mathematical or not, are describable or thinkable in terms of the limited figures of differential equations and smooth manifolds. Deleuze's metaphysics constrains itself by becoming too localized due to its failure to cover the entirety of Being.[18])

42. Post-Badiou, this remains to be demonstrated: a 'proof of concept' for an interesting, novel and compelling philosophical compossibilization of a non-meta-mathematical mathematics. Perhaps more work needs to be done and more conceptual innovation is required. In my own personal attempts, the impasse involves avoiding taking the mathematical definitions, theorems, proofs and frameworks as just similes for some external philosophical conceit. It is not obvious how one may even begin to go about instigating any event of metaontological thought out of, for example, the extremal combinatorics of Ramsey Theory, particularly beyond the often-denigrated route of metaphorical provocation or analogical induction. In my case the difficulty involves moving from the austere formality of the mathematical figure to the crude but profound generality of a philosophical proposition. And to do so while remaining committed to the implications of the 'ontology = mathematics' equation.

43. The unappreciated genius of Badiou's equation is, among others, this diagonalization away from the easy path of metaphor. Badiou's equation means that mathematics is not just a symbol or idiom for ontological ideas. As *precisely* the *immediate* inscription of Being qua Being into thought, mathematics avoids ontologizing indirectly in terms of mytho-poetic symbols that lack rigor or obscure hymns towards some original alterity.

44. Let us however confess that we still have not completely understood the mechanics, politics and ethics behind the employment of metaphorical figures into philosophical thinking. This issue becomes doubly-complicated post-Badiou because philosophy is now understood to be at its core a trans-disciplinary enterprise that, through aleatory movements of commitment and construction, imports truths and vocabularies from other.

45. It is not wrong that normal words and figures become bastardized as they slip between disciplines? Are not all words ossified metaphors?

15 Badiou 2007b.

16 Badiou 2008.

17 The best explication of Deleuze's mathematical metaphysics is DeLanda 2013.

18 The same might be said for Leibniz's metaphysics of the monad, which can be interpreted as being conditioned by his work on the unique prime number decomposition of integers. But to say that Being qua Being is essential prime number, or just number, as the Pythagoreans did, is incorrect.

Jacques Derrida writes that all proper primitive meanings, which are transparent figures, cannot escape becoming metaphorical when placed in philosophical circulation. 'The metaphor is no longer noticed, and it is taken for the proper meaning. This is a two-fold effacement. On this view, philosophy would be a self-eliminating process of generating metaphor. It would be of the nature of philosophy that philosophical culture be a rude obliteration'[19]. Is not truth, as Nietzsche teaches us, just a 'mobile army of metaphors'?[20]

46. I believe much practical guidance and methodological analysis needs to be done, beyond what Cohen's mathematics has given us, on understanding the general process of forcing and of compossibilization as they are implemented by philosophical thinking, while still maintaining and respecting the essential aleatoriness and free subjective sovereignty of that process. Due to certain structural features, the forcing relation differs from the simple logical relation of implication. Philosophical ideas are not logically inferred from non-philosophical truths. 'To force', 'to condition' – this is not exactly the same as 'to cause', 'to influence', 'to inspire' or 'to model'. It is certainly not the same as 'to symbolize' or 'to signify'. And it is not enough to reduce the relation of forcing to the subjective moment of deciding, despite the general phenomenon of bounded rationality in both processes.

Metaontologies of Specific Mathematical Fields

47. Let me end here with a few programmatic notes on the possible construction of a metaontological thought from other mathematical fields. Each of the branches within mathematics, while remaining wholly within a specific subdomain of ontology, concerns itself with specific forms of Being. For algebra, for example, it might be structure or symbolic structure. For arithmetic, it is number and counting. For geometry, metric space or manifold. For calculus and analysis, continuous change or movement. And so on for combinatorics, topology, statistics, probability theory, and so forth. But a mathematical theory also has its own vision and cognitive technology for approaching ontology. Each field within mathematics can be said to supply a unique cognitive machinery

19 Derrida 1974, pp. 8-9.

20 Nietzsche 2012.

CRISIS & CRITIQUE / Volume 5 / Issue 1

for thinking Being qua Being, technologies which often become even more powerful when they cross-pollinate amongst themselves. Descartes's discovery of coordinate geometry brought about a new way to understand space and manipulate it algebraically. Analytic number theory, the merger of analysis and number theory, allowed us to understand the additive properties of prime numbers using the tools from calculus.

48. Each branch of mathematics draws from different human intuitions, cognitive possibilities, and ontological techne for understanding what is means to be. Take for example, the ontological techne of algebra versus geometry, of structural versus spatial thinking. Observe that it is much easier to count the number of sides on a cube by visualizing it in your head (two front and back, two top and bottom, and two left and right, for a total of six) than to plug in some algebraic formula. The visual part of the brain, its powerful Graphics Processing Unit (GPU) so to speak, is evolutionarily well-equipped towards thinking and intuiting about certain ontological issues involving spatiality than others. The mathematician Michael Atiyah speculates that the fundamental reason, 'is that geometry is the least abstract form of mathematics [....] By contrast algebra is the essence of abstraction, involving a dictionary of symbolism which has to be mastered by great effort [....] [G]eometry is that part of mathematics in which visual thought is dominant whereas algebra is that part in which sequential thought is dominant.'[21]. Algebra and geometry provide unique tools for us to access a thought of Being qua Being[22].

49. (This may explain Deleuze's choice of differential geometry as the truth condition for his metaphysics. The concreteness of geometry and the dynamism of calculus, when merged together, provide the most suitable cognitive technology and perfect paradigm for constructing his philosophy of vitalist materialism. With the recent decline of differential equations in favour of statistical and probabilistic methods in applied mathematics and the natural sciences – not to mention the possible rise in a few years of powerful computerized Deep Learning and Artificial Intelligence methods for data analytics - perhaps Deleuze's entire schema can be modified or updated by replacing differential with stochastic techniques. Might

21 Atiyah 1982, p. 179.

22 To use the Lacanian vocabulary, algebra is closer to the Symbolic whereas geometry is closer to the Imaginary realm of psychic phenomena.

the Deleuzian movement of the virtual, whose ontological figure is the infinitesimal movement, be replaced by some corresponding figure conditioned from probability theory or statistics?)

50. A possible diametrically opposite counterpoint to Badiou's axiomatic metaontology would be mathematical fields that, for reasons mathematicians still do not understand, essentially recede from formal and systematic axiomatization. Some of these fields present what we might even postulate to be an essential resistance, a radical withdrawal, with respect to this Will towards Theory.

51. To be sure, many scientific fields, particularly the most empirical ones, withdraw being engulfed by this Will. Due the unavoidability of non-zero error terms, the limits of experimental analysis, and the essential complexity of life and the élan vital itself, biological knowledge cannot help formulate itself as statistical principles instead of formal theories or theorems. Biological research is rarely about big theories, unlike in theoretical physics and abstract mathematics. In an interview, Badiou even went as far as to reject biology as a science[23] and, by extension, a possible domain for philosophical compossibilization.

52. This struggle against formalization is also present in a different form within many fields within pure mathematics, particularly on the 'problem-solving' as opposed to the 'theory-making' style of mathematical research. For the former, the point of mathematics is to build mathematical theories, and problems are only solved in order to understand mathematics better. For the latter, the point of mathematics is to solve mathematical problems, and mathematical theories are built in order to become better at solving problems.

53. Within theory-building mathematics, we might have most of the subfields of algebraic geometry, the Langlands program, and the work of Badiou's mathematical hero, Alexander Grothendieck, with his famous analogy of solving a mathematical problem as being akin to opening a nut slowly by immersing and rubbing it in soft liquid[24]. For Grothendieck, a problem is solved by building the most general theoretical infrastructure for it. The right theoretical perspective must be erected so that the problem could be solved effortlessly

23 Badiou 2006, pp. 235-6.

24 Grothendieck 1985-1987, pp. 552-3.

and naturally. The solution then becomes the most obvious thing in the world and fits naturally into the larger and abstract narrative. The mathematician William Timothy Gowers writes that for theory-building mathematicians, 'it is important for many reasons to build up a considerable expertise and knowledge of the work [...that] other mathematicians are doing, as progress is often the result of clever combinations of a wide range of existing results'[25].

54. Within problem-solving ontology, we might have certain subfields within combinatorics, partial differential equations and number theory. Grand Unified Theories are often lacking in those fields. The main organizing role is played, not by general abstract theories, but the mathematical tricks and tools. Ontology places itself on the side of techne and not theoria. Abstract generalization of specific solutions can only go so far because they are often uniquely tailored to the problem in question. Problem-solving for such fields can only be done on a case by case and ad hoc basis[26].

55. The well-known problem-solving mathematician, Paul Erdös, is also the most productive mathematician of the past century. His oeuvre can be seen as the invention of a series of ad hoc tricks, modified to fit the situation in question, for solving mathematical problems. For the mathematical field of combinatorics, Gowers writes, "The important ideas [....] do not usually appear in the form of precisely stated theorems, but more often as general principles of wide applicability"[27]. The field of graph theory, which deals with the topology of networks, does not progress by formulating and analysing some formal axiomatic system for the notion of the graph. "[T]he basic object, a graph," Gowers writes, "can be immediately comprehended. One will not get anywhere in graph theory by sitting in an armchair and trying to understand graphs better. Neither is it particularly necessary to read much of the literature before tackling a problem [...] the interesting problems tend to be open precisely because the established techniques cannot easily be applied.[28] Graph theory, an ontology that withdraws from formal axiomatization, is a blind spot to theoretical metaontology.

25 Gowers 2000, p. 3.

26 A famous result in number theory, Matiyasevich's Theorem proves that the general class of Diophantine equations in number theory, for example, lacks a universal procedure for solving it.

27 Gowers 2000, p. 5.

28 Gowers 2000, p. 3.

56. If we allow, as viable truth-conditions to be pursued, these different visions and technologies for philosophical thinking, then a new possibility for mathematical metaontology and philosophical truth might announce itself. If we pursue these alternative to the meta- and theory-building mathematics, then the philosophical project of compossibilization might shift towards an enlarged vista. Ontological and metaontological thought becomes not just the composition of large, though constantly reconstructed and deconstructed, edifices. It can also admit within itself the aleatory dialectic of problem-solution-problem-solution – perhaps not unlike what had been proposed by Deleuze's philosophy[29] – instead of the infinite but abstract weaving of a generic truth procedure and a new generic situation.

57. Or perhaps a different phase of Badiouian metaontological thinking will appear, a mathematical metaontology of technology technicity, not unlike what happened to Heidegger's philosophy after the Second World War with the publication of 'The Question Concerning Technology'[30]. In relation to Being qua Being, ontology thereby will be understood not just as a scientific discourse but a technology of Being qua Being.

58. Perhaps then a new condition, technology – or even engineering – could be allowed to supplement science as a domain out of which philosophical compossibilization could be implemented. An interesting possibility, provided that enough subjects would be committed to it philosophically.

29 See Smith 2004, for the best comparative analysis of Deleuze's 'problematics' and Badiou's 'axiomatics'.

30 Heidegger 1977.

Bibliography:

Atiyah, Michael 1982, 'What Is Geometry? The 1982 Presidential Address', *The Mathematical Gazette*, *66*(437): 179-184.

Badiou, Alain 2000, *Deleuze: The Clamor of Being.* U. of Minnesota Press.

-- 2006, *Theoretical writings*, Bloomsbury.

-- 2007a, *Being and Event*, Continuum.

-- 2007b, *The Concept of Model*, re:press.

-- 2008, *Number and Numbers,* Polity.

-- 2009, *Logics of Worlds: Being and Event, Volume 2*, Continuum.

Cohen, Paul 2008, *Set Theory and the Continuum Hypothesis.* New York: Dover.

DeLanda, Manuel 2013, *Intensive Science and Virtual Philosophy*, A&C Black.

Derrida, Jacques 1974, 'White mythology: Metaphor in the text of philosophy', *New Literary History*, *6*(1): 5-74.

-- 1993, 'Structure, sign, and play in the discourse of the human sciences', In J. Natoli & L. Hutcheon, eds., *A postmodern reader*, SUNY Press.

Gödel, Kurt 1931, *Über Formal Unentscheidbare Sätze der Principia Mathematica und Verwandter Systeme, I. Monatshefte für Math. u. Physik 38*: 173-198.

Gowers, W.T. 2000, 'The two cultures of mathematics', in V. Arnol'd, ed., *Mathematics: Frontiers and Perspectives*, American Mathematical Society.

-- 2006, 'Does mathematics need a philosophy?', In R. Hersch, ed., *18 unconventional essays on the nature of mathematics* (pp. 182-200), Springer.

Grothendieck, Alexander 1985-1987, *Récoltes et semailles. Réflexions et témoignage sur un passé de mathématicien*, *Univ. Sc. et Techniques du Languedoc, Montpellier*

Heidegger, Martin 1977, *The question concerning technology*. Harper & Row.

-- 1978, *Wegmarken,* Frankfurt am Main: Vittorio Klostermann.

-- 1996, *Being and Time*, SUNY Press.

Meillassoux, Quentin, 2010, *After Finitude,* Bloomsbury.

Nietzsche, Friedrich 2012, *On Truth and Lies in a Nonmoral Sense*, Aristeus Books.

Quine, Willard V. 1948, 'On What There Is'. *The Review of Metaphysics*: 21-38.

Russell, Bertrand 2013, *Mysticism and Logic*, Courier Corporation.

Smith, D.W. 2004, 'Badiou and Deleuze on the Ontology of Mathematics', In P. Hallward, ed., *Think Again: Alain Badiou and the Future of Philosophy*, Continuum.

Voevodsky, Vladimir 2013, *Homotopy type theory: Univalent foundations of mathematics*, Princeton: Institute for Advanced Study, The Univalent Foundations Program.