Phenomenology of Value: Badiou and Marx

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Abstract: *The notion of value in Marx's work is unique in that it resides in the space between an objective fact and ideology. It is both a source of misrecognition and of theoretical clarity for Marx's overall project. In this text we make speculative use of Badiou's phenomenology, developed in his Logics of Worlds, to analyze value. Our thesis is that value is a phenomena which is made of several logical components which were elucidated by Marx, but that Badiou's framework can show a new way in which this phenomena is immanently constructed. Our aim is to show how this is both more objective and more coherent with modern mathematics than previous interpretations.*

Keywords: *phenomenology, value, Badiou, Marx, fetishism*

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Introduction

The following work argues that Marx's version of the law of value can and should be formulated in the language of Badiou's phenomenology. Most expositions about the law of value usually focus on its explanatory force or its empirical undecidability. This is because, as a foundational question in Marxist political economy which continually attempts to establish itself as scientific, its value seems to reside in validating (or invalidating) Marxist political thought as such. This text takes a different approach: rather than attempt to prove or disprove the law of value, we ask what sort of questions can be possible on its basis. In other words, what does a world where this law is operative look like?

It is important then to qualify in what sense value (as delineated by Marx) can exist within a world, which is where Badiou enters. We show how his philosophy can be utilized as a tool for extracting the important features of our question and transforming them into new vantage point on the theory of value. Specifically, we wish to show that the phenomenology of Badiou is a framework suited for studying value because value is phenomenal in the strict sense. To give that statement its full import, we need first to construct a bridge between Badiou and Marx.

Fetishism

The central term of this bridge is that of *fetishism*, the name given by Marx to a certain perspectival error engendered by capitalist social relations. Simply put, as soon as a product of labor enters the market, its value becomes unhinged from the labor which produced it. The value of a product becomes a matter of comparison with other products, even though its true source lies in the productive *process*. There is then an

incommensurability between the market and the factory insofar as value is "recomputed" in the passage from one to the other. Based on this, it is permitted to say that there exists two "worlds" in which value exists, that of production and of circulation, and the problem of fetishism lies in the disjunction between their respective modes of value (use and exchange). The crucial point is that production is already caught in the network of exchange, since it is effectively comprised of an assemblage of commodities (the means of production) including labor power, itself a commodity.

The second order problem is this: knowing about the true state of things, that value is actually created by the worker, does not at all affect fetishism. This is because fetishism articulates itself at the level of economic activity - we behave *as if* a commodity has value in itself and not because it was produced, regardless of how enlightened we are of the actual situation. On the other hand, fetishism is not impossible to discern, but is rendered palpable by a certain line of thought, namely Marxist critique. However, it is an illusion which does not dissolve even after we've uncovered it, which is why an "objective phenomenology" that does not rely on subjective impressions is needed.

For Marx, value *necessarily* appears bifurcated, not due to missing information, but because of a truly ontological split. To name it fetishism is not to diagnose a psychological defect, but to name a really existing "component" of value. This is a crucial point that perhaps many Marxists would not agree with: the value-form would not be what it is without fetishism. The remainder of our argument rests on this point. Our thesis is that, in Badiou's terminology, fetishism is a *real atom* of (the appearance of) value. For Badiou, *real atoms* are one side in a relation between phenomenology and ontology, between value's appearing and being. Our statement, formulated in Badiou's materialist framework, implies that value is a phenomena and is therefore supported by being(s) which can be analyzed mathematically.

Before going into detail on this, let us examine in what sense this corresponds to Marx's own definition:

"The bodily form of the commodity becomes its value form. But, mark well, that this quid pro quo exists in the case of any commodity B, only when some other commodity A enters into a value relation with it, and then only within the limits of this relation. Since no commodity can stand in the relation of equivalent to itself, and thus turn its own bodily shape into the expression of its own value, every commodity is compelled to choose some other commodity for its equivalent, and to accept the use value, that is to

say, the bodily shape of that other commodity as the form of its own value."[1]

The value form is peculiar - it only becomes visible once two commodities enter into a specific relation. It is necessary to view commodities from the standpoint of this relation in order to see how one commodity must play a special role, and that the relation is one-sided, asymmetric. Marx insists that a commodity can never embody its own value, but only the value of another. For him, value as such is comprised of at least two parts, the relative form and the equivalent form[2]. The more developed value becomes, the more these two forms stand in contrast. This process eventually leads to the appearance of money as the "universal equivalent".

What is important for us in these initial moments of Marx's construction is that the successive forms of value, while embodied by actual commodities, are not reducible to them. The interplay of these forms reveals more than what each started with - it reveals value as a social relation which is, in an important way, *indifferent* to the particular constitution of the commodities which support it[3]. This indifference is what allows us to pass from the local appearance of value to the global one, a passage amounting to the emergence of a common, social substance.

The true nature of this social substance is the question posed by Marx when he discusses fetishism. Namely, we begin to treat social relations between processes of labor as a property of their products:

"As a general rule, articles of utility become commodities, only because they are products of the labour of private individuals or groups of individuals who carry on their work independently of each other. The sum total of the labour of all these private individuals forms the aggregate labour of society. Since the producers do not come into social contact with each other until they exchange their products, the specific social character of each producer's labour does not show itself except in the act of exchange.

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1   Marx 1867 p. 38

2   The passage continues: "Since the relative form of value of a commodity – the linen, for example – expresses the value of that commodity, as being something wholly different from its substance and properties, as being, for instance, coat-like, we see that this expression itself indicates that some social relation lies at the bottom of it. With the equivalent form it is just the contrary. The very essence of this form is that the material commodity itself – the coat – just as it is, expresses value, and is endowed with the form of value by Nature itself. Of course this holds good only so long as the value relation exists, in which the coat stands in the position of equivalent to the linen."

3   For more on the peculiarity of this social relation, see Tupinambá 2014, pp. 318-326

In other words, the labour of the individual asserts itself as a part of the labour of society, only by means of the relations which the act of exchange establishes directly between the products, and indirectly, through them, between the producers."[4]

Value only appears in the relation between two commodities, in the context of exchange, and therefore obscures the relation between producers which is its true source. It is not simply that we do not recognize the organizational dynamics between workers in the end product. More importantly, we act as if these dynamic social relations are that of commodities themselves, as if they have a (social) life of their own. This abstract sociality of commodities serves to "mediate" the relations between people and becomes the measure of society.

As Marx emphasizes, this displacement is doubly important in the case of labor, which is now only counted as "the labor of society" *through* the relations between its end products. Labor, which begins as the source of value, ends with its own value, as if it is also another product of labor. In this way, the value-form effaces its own history.

While the relations in Marx's famous definition of fetishism (that originate between people but are later displaced onto things) are "real relations", what has to be explained is how they take on an illusory, or more precisely, phenomenological form. Again, this form is objective and can therefore be analyzed. This is the entire aim of the critique of political economy: to isolate and examine the effects of this form of appearance[5]. The challenge is that, like the unconscious, analyzing the form of value includes analyzing the very way that it tries to hide itself.

Even though we are dealing with appearance here, the act of exchange reveals facts about the *being* of capitalist society. By virtue of treating value as phenomenon, we are able to give Marx's Capital its proper place, as the *ontological* exposition of successive layers of the social substance in a mode of intercourse organized by the commodity form. Value is comprised of phenomenological substrates which Marx pulls apart and puts back together. What Badiou makes rigorous here is the a-subjective, logical character of this process. If we follow Badiou's materialist claims, then we should be able to show that each layer of the

definition of value corresponds to a level of being. Therefore, in order to justify our claim that fetishism in Marx is a real atom, we need to identify this ontological counterpart.

An Ontology of Value?
We said previously that fetishism arises from social activity, the way we treat commodities during exchange. Can this be generalized to value as such? Is value defined by the circulation of commodities? One could argue that, without exchanges happening, there would not be value as such, only use-values. Yet, the law of value imposes a different thesis, that one of the commodities which is currently circulating, labor, is actually the true source of value - which entails both that there is a paradox in the commodity-form and that use value is itself conditioned by value, rather than something purely heterogeneous to it. To confront this question will bring us to the heart of the matter regarding the ontology of value.

What is special about Marx's notion of value opposed to Smith, Ricardo, et. al? All labor theorists conceive labor-time as a certain "boundary condition" for the exchange-value of goods. Namely, although prices can fluctuate given external conditions (scarcity of resources, accidental conditions of production, consumer preference), there is an underlying determination by the time it takes to produce the product on the value at which the product trades. Marx simplified and made rigorous the terms of this relation, but all in order to put forward a question which he claimed to have been missed by his predecessors, namely, how this deterministic relation *comes to be*. As he states in *Poverty of Philosophy*:

"Economists express the relations of bourgeois production, the division of labour, credit, money, etc., as fixed, immutable, eternal categories. M. Proudhon, who has these ready-made categories before him, wants to explain to us the act of formation, the genesis of these categories, principles, laws, ideas, thoughts.

Economists explain how production takes place in the above-mentioned relations, but what they do not explain is how these relations themselves are produced, that is, the historical movement which gave them birth. M. Proudhon, taking these relations for principles, categories, abstract thoughts, has merely to put into order these thoughts, which are to be found alphabetically arranged at the end of every treatise on political economy. The economists' material is the active, energetic life of man; M. Proudhon's material is the dogmas of the economists. But the moment we cease to pursue the historical movement of production relations, of which the categories are but the theoretical expression, the moment we

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4   Marx 1867 p. 48

5   What should be added is that this analysis only takes place from an engaged position. Badiou's notion of an objective appearance does not exclude the possibility that one's subjective position allows a clearer view of a given phenomena. But it *does* exclude the possibility that subjectivity is responsible for constituting or synthesizing phenomena (see Badiou's critique of Kant in Badiou 2006 pp. 231-241).

want to see in these categories no more than ideas, spontaneous thoughts, independent of real relations, we are forced to attribute the origin of these thoughts to the movement of pure reason."[6]

According to Marx, it seems the ontologization of value is precisely a bourgeois economic invention, assuming the "ontologization" is the same as "de-historicization". Whereas other economists take for granted value as a category, Marx wants to show that value appears in developed and not-so-developed forms. These forms are related in a way (denoted by *aufhebung*) such that one form never completely replaces the other. Rather, they together comprise an ever-developing logical-historical space. So is any description of value in Badiou's set-theoretic ontology impossible? Can the value-form be modeled mathematically? This seems at first glance to be completely at odds with Marx's strategy of infusing economic theory with temporality. There is indeed a tension between Badiou's commitment to a formal exposition of appearance and the above quote. Our aim is therefore to show that Badiou's system is not only capable but inherently suited to model Marx's logic.

Introducing the historical parameters of value as somehow *constitutive* of it seems like a very unscientific move when compared to the mathematically rigorous methods that reinforce modern economics. If history is that subject which most resists objectivity, it is mathematics which serves as the model discipline for studying infinite, impersonal reality. Marx's question therefore seems like a *regression* when it posits a "coming to be" of the concept of value: it opens political economy up to the guesswork of history. But this work of exposing the scientific field to historical analysis has a philosophical-critical relevance. Althusser named it the arrival of class struggle in philosophy. Namely, if we take class struggle as an objective fact, we are then permitted to ask the question: what does a particular domain of (scientific) knowledge look like from the point of view of this fact?

This invention of an "objective perspective" from which to view science is fundamental. It supposes both that history can be examined scientifically and science examined historically. It is here that we find certain affinities between Marx's critique and Badiou's philosophy. The latter is perhaps the most systematic exposition of a logic of appearance since Hegel's and is based on the premise that *appearances are objective*. For Badiou, there is no need for a subject to which things appear, since these appearances have definite relations among themselves independent of whether they are perceived. This leaves open the question of why some things appear and others do not. Badiou's notion of *event*

from his earlier work formalizes this question - among its effects, the event is what opens up a region of appearance for a subject[7].

Taking up fetishism again, we can identify the perspective, whereby this cleft in value is visible, as the standpoint of class struggle. Another way to approach our question, then, is to *assume the mistake* of treating circulation as the being of value. This is the correct move insofar as fetishism is part of the historical parameters of the object under scrutiny. We should not simply do away with it, but rather treat it as a valid (i.e. localized) component of the appearance of value. This brings us inevitably to mathematical formalism.

According to Marx, the difference between the normal value-form and the capital-form lies in the *use* of the commodity. Of the multiplicity of forms of use which unfold in history, the one which is capable of producing surplus value is unique, since it introduces an apparent autonomy to value. We act *as if* value were self-generative, as if value makes more value, but we know it is actually labor behind it. This self-referential "illusion" reaches a point where it becomes indistinguishable from a natural process. Marx's method is to examine the this process independently of the thinking subjects who carry it out. This is his account of the movement from simple circulation to capital:

> "The first distinction we notice between money that is money only, and money that is capital, is nothing more than a difference in their form of circulation.
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> The simplest form of the circulation of commodities is C-M-C, the transformation of commodities into money, and the change of the money back again into commodities; or selling in order to buy. But alongside of this form we find another specifically different form: M-C-M, the transformation of money into commodities, and the change of commodities back again into money; or buying in order to sell. Money that circulates in the latter manner is thereby transformed into, becomes capital, and is already potentially capital."[8]

7   And the subject *is* insofar as it retains a certain fidelity to this event. This definition of subject raises an entire problematic of "what it means to be faithful", specifically when we consider that a subject is not an individual but could just as well be the scientific establishment, an artistic movement, a couple in love, or a group of militants. Yet it offers us a way of conceptualizing how class struggle seems to appear in so many disparate areas once we are engaged by Marx's thought.

8   Marx 1867 p. 104

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Here we have an entire exposition of phenomenological logic which begins with the question of the "form of circulation", that is, how circulation *appears* in a two-fold manner. First, we should note that **C-M-C** and **M-C-M** rely on the same underlying sequence: the one which rules that **C** and **M** are always adjacent. The difference lies entirely in the subordinate conjunction "in order to". The first form still terminates with the consumption of the commodity, its utility lies in being able to procure a new commodity from the previous one. The second form of circulation is not simply a different use of money, but tells us something new about money itself, that it can subordinate or subvert the chain it belongs to.

In other words, circulation has shown us an ontological fact: that money is "already potentially capital". Money does not historically appear as value-producing, but once the logical passage between "selling in order to buy" to "buying in order to tell" is complete, something is nonetheless revealed about money's origins. We can speculatively translate this passage to Badiou's framework with the idea of a "phenomenal component"[9]. Recall that we discussed earlier the relational character of value - it is always supported by at least two commodities, one of which takes on the "equivalent form". The role of this equivalent is to effectively embody value itself *for* the other commodity. Badiou allows us to discern what sort of incarnation is at stake here when he suggests that each phenomena can - to a greater or lesser degree - be identical to another, and that this degree is determined by the "phenomenal component" in question.

This requires an inversion of perspective which is properly dialectical. An example of this would be the dramatic element of a play: it is one thing to say that a scene in a play contains drama, another to say that this scene belongs to the drama of the play. In the former case, "drama" is simply a property of a something (a scene or performance), but in the latter, it becomes the measure of everything else. This latter measure is precisely the function of the phenomenal component, which assigns to every ontological element a degree of belonging to it. This is the function which money takes on as it becomes the embodiment of value for all other commodities.

Money in its character as universal equivalent assigns a certain degree of existence to other commodities, yet it is also just another commodity. In other words, it is an immanent, self-regulation of the

commodity-form. As consumers, this seems intuitive. We behave on some level as if two commodities are the same because they have the same price, despite their ontological differences (e.g. a university education versus a new house). Inversely, we can treat two very similar commodities as different because of their price (e.g. a meal at a high priced restaurant versus something cooked at home).

This becomes more complicated when we include the production process itself as a commodity: the working hours of one laborer can be valued drastically different than another, which generates the perception that one group of laborers *is* drastically different than the other[10]. At this point, a certain closure of the logical space of value occurs, since the very production of value is captured in the process of circulation. This closure has consequences, among them, the abstraction of labor. To use terminology, there comes to exist a *map*[11] between the diversity of productive processes and money. As a result, all labor becomes countable as discrete work-hours, etc. and surplus value becomes calculable.

The sequence **M-C-M** should be read as a formula of the commodity form itself. There is a difference in value between the first instance of **M** and the second, attesting to the fact that **M** is not a variable. There are two usual approaches to this apparent oversight. The first is to rewrite the formula as $M_1\text{-}C\text{-}M_2$ such that M_2 minus M_1 is profit, and to take this to be a formula for surplus. The second is to regard this formula as a chronological sequence, or altogether as pre-mathematical, and to focus instead on the argument that Marx makes. In other words, dialectics or mathematics?

With category theory in general, and Badiou's system specifically, there is another approach which can preserve both aspects. Namely, we can conceive of **M** as both the *domain* and *codomain* of a function **C**. **C** takes a certain amount of money as input and returns a different output of money, but both the input and output belong to the same set. Likewise, in simple circulation, **M** is the function and **C** is both the domain and codomain[12].

The Category of Commodity Circulation and Capital

Badiou posits mathematics as ontology, and the stakes of this

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9   It is important to clarify why it is not enough to posit that money is the transcendental. While it may be true that money is the transcendental for the world of commodities, what we are considering is the world where the value-form is itself analyzable on the basis of class struggle. In this world, the value-form is an object developing locally. In this (Marxist) context, money better models the phenomenal-component, since it is only a sub-phenomenon of the overall value-form.

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10   This is, of course, linked to the question of racism and sexism. Class consciousness is the hypothesis that the identities and differences between laborers, which was previously regulated by this money-component of the commodity-form, can be changed.

11   Meaning that all work has a price, and correlatively, that labor processes relate to each other via their prices.

12   Such functions are called endomorphisms.

position should always be referred back to the foundational tensions of mathematics itself. Category theory, initially developed to formalize the link between algebraic objects and topological spaces, has gradually risen to the candidacy of a foundational theory for all mathematical work. A major turning point in this rise is the reformulation of set theory in categoric terms[13]. This is relevant to us insofar as the statement "mathematics = ontology" may raise questions as to the border of ontology and phenomenology. However, to say that one is a specific case of the other is to mistake the true import of category theory for Badiou. Already in *Being and Event*, Badiou states that "ontology is a situation", which indicates that it is a *being-there* and not a being. The inscription of set theory in the language of categories is not a generalization of the work of thinking being, but rather its localization. This implies that Badiou, like Marx, allows us to think the value-form in an extrinsic, historical way[14]. We should keep this in mind as we now develop our mathematical reading of the commodity circulation.

Some rules follow which force us into some technical work. First, a function never maps an input value to more than one output value. Second, every value in a domain must have an output value. Therefore, in simple circulation we need to ensure that one commodity cannot be transformed by money into multiple commodities, and every commodity must be able to be transformed.

In the first case, while it is true that one can turn a single commodity into multiple (e.g. selling one commodity and using the money to buy multiple commodities), this implies that the commodity being sold itself can be portioned into parts corresponding to the multiple commodities being bought. A single exchange of a more valuable commodity for several less valuable ones can be written as multiple exchanges of parts of the former with the latter. This is due to the fact that money is divisible and therefore enables the partitioning of the commodities that it represents. In the second case, a commodity can always be transformed since any commodity can potentially be bought or sold.

So our condition holds for simple circulation (C-M-C). What is required in the case of capital (M-C-M) is to do the same exercise taking the commodity as the function and money as the set[15]. The condition that each input must have only one output is satisfied by the fact that if I buy something now, I can sell it again later. The other condition that all inputs

must have an output is achieved trivially by the fact that money can always be exchanged for itself.

We now need to generalize these set theoretic definitions. In order to define a category, we need the following: a collection of objects and arrows satisfying composition, associativity of composition, and identity. This can be done in a simple way if we identify a single object, called **M** and an arbitrarily large number of arrows designated by C_1 to C_n[16]. The object is simply the money set we defined previously, but stripped of its interior, since objects do not "contain" anything. What we care about is simply that it is both the source and target of the family of **C** arrows, which corresponds to individual commodities[17].

Composition is satisfied by the basic fact of circulation - we can buy a commodity at a price, sell the commodity later, and use the money to buy another commodity. We can repeat this sequence indefinitely, each time generating a subsequent amount of money (more if it is a profitable sequence and less if it is unprofitable). The key point is that such a sequence, a "business", is itself a commodity. This realizes the formal condition of a category. Composition amounts to our ability to replace any number of enchained arrows with a single arrow, such that the initial source object and the final destination object of the sequence is now the source and destination of the single arrow. The second condition, that of associativity, simply requires that we can do such replacements in whatever order we want, and it will always arrive at the same single arrow.

Take the following example:

where is the money set and where is the indexed family of commodities. This is usually written as where is arrow composition. Associativity implies that - which means we can perform the composition on first and then or first and then , and the result is the same.

Generally, with categories one cannot change the order of the enchainment, but here composition is *both* associative and commutative. Finally, the identity condition requires the existence of an arrow C_i which, when composed with any other arrow C_n, is the same as C_n. This would be a commodity that I can buy to ensure that I can later sell another

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13   For more on the "elementary theory of the category of sets", see: https://ncatlab.org/nlab/show/ETCS

14   This was only made clear to the author recently in a debate with another member of the Subset of Theoretical Practice.

15   Here we assume the money-set is isomorphic to the set of natural numbers.

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16   Also called a monoid category. Subsequently, we can define a *circulation monoid* and a *capital monoid*, depending on which of M and C is the object and arrow. Monoids can also can be defined in terms of a set of elements and a binary operation which combines those elements. In our case, the elements are commodities and the combining operation is the *composition of exchanges*.

17   Given a monoid category, we can define its arrow category to be one where each arrow is now an object. In the case of capital, the arrow category would be the category of commodities and exchange-arrows. This is a good starting point for further research.

commodity for the same price I bought it for. Insurance is an example of such an identity commodity.

One could argue that these above conditions can only be "satisfied formally", since the actual market value of any commodity (including money itself, which only exists as regional currency) is changing constantly. Therefore, there could arise situations in which there is no commodity which preserves identities, or money within a country becomes worthless. But the point of such an ontological analysis is precisely to discern the being of a commodity, which is also to discern what it means to diverge from this being. To put it another way, there is nothing which guarantees that the actual market obeys its own ontological parameters. So in response, we could argue that Marx's method combined formalism and historical analysis to identify in capitalism the tendency to generate crises which undermine its functioning. For more on this, see the last section of this text.

Given the above description of the passage between simple circulation and capital, we are now left with the question of its appearance in a world. By definition, beings always appear *locally* to a world, possibly in many worlds, whereas an ontological description is global[18]. Badiou demonstrates that appearance does not simply reflect being, but also effectively alters it. For us, this is the key to grasping how the various forms of value we have analyzed take their place in the world *after* Marx.

The Greater Logic
A full exposition of Badiou's system would be beyond the scope of this text, but it is already worth pointing out a few intersections with Marx's critical method. First, the existence of objective appearances also supposes an objective perspective or framework from which to analyze them. For Marx this is class struggle, but for Badiou it is category-theoretic logic, a framework developed by analyzing the most general definition of transformation in mathematics. Incidentally, the early Marx[19] also arrives at the notion of class struggle by analyzing transformations of an arguably more general nature[20], namely labor, and how private property affects its distribution in society.

We begin with Badiou's formal definitions of object[21], world and transcendental. These will be necessarily brief, but we will develop the intuition behind them in latter part of this section and the next one.

First, a phenomenal object is a multiple whose elements are indexed by the transcendental of a world such that any two elements are assigned a degree of identity. Such a thing can be written with a pair of terms **(M, Φ)** where the **M** is multiple and **Φ** is the indexing function. We say that **Φ** has **M** as its domain and a transcendental **T** as its codomain since it takes any two elements of **M** and returns a degree of **T**.

Next, a world **W** is comprised of objects as defined above, one of which is the aforementioned transcendental. This world is characterized by the degree to which we can differentiate its objects, that is, by the relation between its objects and the degrees of its transcendental. This assigning of degrees is performed by **Φ** and is unique to each object within **W**. That is to say, the same multiple **M** may appear in multiple worlds, but in each one its indexing will be different. Every world is supported by an "inaccessible cardinal"[22], that is, a non-denumerable set of all parts of all objects. In other words, a world contains infinite objects but is nevertheless not exhausted by this infinity which is still "accessible"[23].

This leads us to the definition of a transcendental **T**, which is another object in the world, but comprised of things called "degrees". These degrees have an internal relation defined by three properties: *reflexivity*, *transitivity*, and *antisymmetry*, or what is called a *partial order relation*[24]. This allows us to speak of greater and lesser degrees, though not in all cases, since two degrees can be entirely unrelated. All degrees, however, are related to at least the minimum degree . Additionally, given any two degrees, one can produce a new degree via the conjunction operation . The conjunction of two degrees gives us the value of the lesser degree. Therefore, for any degree a. Given a subset of degrees of **T** called **B**, we define the *envelope* Σ**B** to be the smallest degree larger than all degrees within the subset. is distributive over the envelope, such that .

18   To be more precise, an ontological description appears locally in the world of mathematics, but is global or "extra-worldly" in its scope.

19   See Marx 1844, p. 32

20   Indeed, it can be argued that Marx discovers in capitalism the very origins of abstraction which appear later in mathematical thinking. See Sohn-Rethel 1978. The intersection between this and Badiou's work remains an important area of research.

21   Within category theory, there is a fundamental in-difference between object and transformation, insofar as they are interchangeable depending on the category in question.

22   See Badiou's "second constitutive thesis" from Badiou 2006 p. 317 and pp. 345-352

23   The infinity of objects is a denumerable infinity whereas the set of all parts of all objects is non-denumerable.

24   Reflexivity: a degree is always related to itself.
Transitivity: given degrees **A**, **B** and **C**, if **A** is related to **B** and **B** related to **C**, then **A** is related to **C**.
Antisymmetry: if **A** is related to **B** and **B** related to **A**, then **A** and **B** are the same degree.

This internal relation is then "projected" into the objects within a world, insofar as they are defined by a mapping to **T**. Let us define a map **C** for every degree **t** of **T**, such that $\Phi(C_t(M_t)) = t$[25]. Intuitively, C_t filters the object for the parts[26] of it which have the same degree of existence **t**. The family of C maps, one for each degree of **T**, are therefore also related in the same way as the degrees of the transcendental[27]. These parts are what we previously called the phenomenal components and can be arbitrarily granular, such that there could be an infinite family of such sub-components. The granularity is controlled by the transcendental degrees, but also by Badiou's materialist thesis about *atoms*, which following our intuitive definition indicates that the upper limit of granularity is determined ontologically.

In mathematical terms, let us fix some element of the multiple **M** in question and call it **c**. The atom A_c assigns a degree of similarity for each appearance-relation between **c** and all other elements of **M**. In other words, A_c tells us how similar **c** is to its neighboring elements, and this similarity measure is again expressed in terms of transcendental degrees. Note that "neighboring" and "similar" here should be emptied of semantic content: it could mean the elements of **M** are colocated in space, time, or within some other metric(s) (e.g. color, hardness, loudness, etc.) - what is essential is that an atom of appearance makes possible a logical ordering of the elements of **M**, and by extension, the phenomenal object.

There is in fact an ordered relation between Φ, C_t, and A_c: C_t is a restricted version of Φ since it only maps to a single degree **t** of **T**, and A_c is a restricted version of both C_t and Φ, as it is the phenomenal component which is identifiable by an element of **M**. Put another way, the sub-object obtained by C_t may contain multiple parts of a phenomena, but when it only contains a *single* part corresponding to an ontological element of the underlying multiple **M**, it is atomic. Or in terms of Badiou's earlier problematic: the atom is the manifestation of the One in appearance[28].

In this sense, there exists an identification between (phenomenological) atoms and (ontological) elements. Appearances are "real" because, at their bottom, they are identifiable as discrete

ontological units: this is Badiou's first "materialist thesis"[29]. These atoms effectively express a differential structure on appearance - each atom can be conceived of as the smallest real unit of a given phenomenon insofar as it is nothing but the assigning of difference to all other atoms.

Furthermore, just as sets in Badiou's ontology are only ever comprised of other sets, the phenomenological object is only ever comprised of these gradients of identity. To illustrate this, Badiou uses a host of examples ranging from paintings and music to political rallies and scientific experiments[30]. Appearance is redoubled, for what seems like a purely subjective experience (e.g. looking at a painting) is itself conceived as an objective movement through what we could call the object's contextual space (e.g. the world of the painting).

Badiou performs this reduction of qualia to quanta via the transcendental indexing in order to demonstrate two things. First, it reinforces his claim on objectivity, that appearances are not perceptions requiring a subject, but only rely on their immanent[31] constitution and the transcendental of their world. Second, it allows for the reconstruction of phenomena on the basis of changing these relations, as opposed to changing our perception.

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25   In other words, C_t maps a sub-multiple M_t **M** to **M**.

26   This sub-object can also be considered a *fiber* of **t** under **C**.

27   The maps in C are partially ordered with respect to inclusion.

28   Badiou 2006 p. 248

....

29   Ibid p. 220

30   Any artistic work, for example, might be static in the sense that it was finished by its creator, but upon closer inspection, it reveals to us a host of ambiguities, contradictions and tensions. In short, the work exposes a logical space for thought to move in. What is important is the way that certain artistic arrangements capture something real and essential about appearances. Badiou offers us a rigorous insight into this process. For him, what we perceive at first to be the "finality" or "this"-ness of an artwork is actually the minimal stable foundation upon which we can think a world.

31   We leave out a discussion of inter-object relations which, by definition, leave the the internal relations of an object unaffected. For more, see the section on relations in Badiou 2006 pp. 335-339.

To recap:

Defn. object

(M, Φ) : multiple with an indexing function

Defn. sub-object

$(M\alpha, \Phi\alpha)$: $M\alpha \subseteq M$, $\Phi\alpha$ is "compatible" with Φ, and $\alpha \in T$

Defn. indexing function

$\Phi: (a, b) \rightarrow T$ where $a, b \in M$ and T is a transcendental

Defn. atom of appearance

$Ac: (c, a) \rightarrow T$ or alternately, $c \times M \rightarrow T$ where $c \in M$

Defn. transcendental

$T: (\mu, \cap, \Sigma, \leq, S)$ such that

$S \subseteq W, (S, \leq)$ is a partially ordered set

μ is the minimum, $\cap$ is conjunction, Σ is envelope

Defn. world

W : collection of all objects including the transcendental

On the one hand, atoms ground appearance in being by marking its minimal units. On the other, it is the transcendental that guarantees the logical coherence between the parts of an appearance. Badiou's name for this coherence is "real synthesis". Three conditions must be satisfied: *order*, *localization*, and *compatibility*. First, atoms each have a degree of existence determined by **,** namely where **a** is the atom**.** Given any two atoms, the indexing function again gives us a measure of similarity. If the degree of existence of an atom **a** is equivalent to its similarity with another atom **b**, then . This allows us to order atoms on the basis of the order relation on **T**. Second, given a degree **d** of **T**, one can define the "localization" of **a** on **d** as the degree of existence of **a** conjoined with **d**. Recall that conjunction of two degrees produces a third (not necessarily different from its factors). This third degree is that of the existence of a new atom produced by the localization (again, not necessarily different from the previous atom). Finally, the compatibility condition states that two atoms are compatible whenever their respective localizations are the same, that is, the localization of **a** on existence of **b** is the same as the localization of **b** on the existence of **a**. Trivially, an atom is always compatible with itself, but we can start to "collect" different compatible

atoms to form larger sub-components of our object. If all three conditions are satisfied, we are able to form the *envelope* of multiple atoms of the object, merging them together[32].

One can envision a process whereby appearances are divided and subdivided by the maps of C indexed on **T** (defined above), until it reaches a halting point where there exist only atomic constituents. Synthesizing these constituents, the original appearance is then re-assembled by virtue of the properties of the degrees and the possibility of enveloping pairwise compatible pieces. We can then raise the question of the different reconstructions that are possible. It can be shown that, for a given transcendental and indexing function, the reconstruction of an object is unique[33].

The Site and its Consequences

Because appearances are objective, they do not need to reveal themselves to an individual subject all at once. There can be appearances which never appear to an actual person, just as there could be thoughts which will never be thought[34]. But this is the opposite of saying that change is impossible, that everything which can be already is. On the contrary, Badiou's vision of the world is a formal apprehension of unceasing change, where some things forever disappear and others reappear. From this interminable flux, thinkable traces of an event can be extracted and bound together, which is the work proper to the "faithful subject". To be capable of thinking this labor in the Marxist context is one of the true political and philosophical projects today.

Badiou delineates four forms of change: modification, fact, weak and strong singularity. Modification is an adjustment of the intensities of objects in a world, such that it leaves the transcendental itself intact. It is the "natural" variation of the world. However, for true change to occur, the being of appearance must itself be counted as an indexed element. Such a being is named a *site*. Ontologically, this being violates the axiom of foundation of ZF set theory, since it is formally a set which belongs to itself. When it appears, the site can either have a maximal or non-maximal degree of appearance (formally, or), named a singularity or a fact respectively. A singularity, likewise, can be divided by whether it produces a maximal or non-maximal consequences. A

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32 See also the *fundamental theorem of atomic logic* from Badiou 2006 p. 263

33 This is, roughly, the uniqueness condition of a *sheaf*. The transcendental "generates" topological properties of the world insofar as its degrees behave as open subsets under inclusion. The indexing of a multiple M produces "sections" of M corresponding to the transcendental degrees. Sheaves (and more generally, pre-sheaves) consist of these sections.

34 These are traces of Badiou's Platonism - that is, Plato filtered through axiomatic set theory.

maximal consequence is defined by making the in-existent element of an object appear, that is, by producing a permanent change in the transcendental which regulates appearances. Likewise, something which appeared previously in a non-minimal fashion must sink to a minimal value of appearing (in order to formally satisfy the structure of the transcendental).

Given this, it is permitted to say that Marx's critique is a *faithful* thinking of how labor is an evental site in the capitalist world[35]. His analysis of capital brought to light what was previously occluded in the value relation, namely, the figure of the proletariat. In the deprivation of the worker, the very being of value appears in contrast to the appearance of surplus value. Badiou identifies the appearance of a site with its disappearance, such that it only leaves a trace in the form of the consequences on other objects. His own example is the *factory*, which is a site precisely because in its disappearing, it renders class struggle visible[36]. He says:

> "Letting myself be guided by these two finds of classical Marxism, the void and the factory, I propose the following thesis: in modern historical presentation, the factory is the event par excellence, the paradigm of the multiple at the edge of the void."[37]

To use the terminology from Being and Event, while the factory as an economic entity is "counted as one" by the State, the workers who constitute it are not a part of this counting. In the capitalist world, the factory is recognized only insofar as it is a *company*, which obfuscates the relations of workers internal to it. In ontological terms, the company is a *singleton* set - it has only one element, generally the head of the company. It is a representation which is then countered by a second representation, the union. Badiou argues, however, that the conflict between these representations occludes the essential problem, that the worker cannot be presented. Seen from the "inexistence" of the worker, both unions and the company are figures which make the "factory as event-site" disappear.

> "Let us say it plainly: if the factory is the paradigmatic event site of our societies, it is because the event within it is strictly speaking impossible without the collapse of the site as one. The

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35   In recounting his own intellectual trajectory, Marx mentions as one of his formative moments his uncomfortable realization of the material conditions of the poor in Prussia (specifically regarding the ruling on theft of wood). See preface to Marx 1859

36   Badiou 1986

37   Ibid p. 172

factory event, since it makes exist the very thing whose inexistence sustains the one-of-the-factory, that is the workers. The factory is this exceptional place in which the charge of singularity is such that to even partially deploy it within presentation one ravages the count, in the irruption of the void which the count exiled and whose errancy it simultaneously concentrated."[38]

The singularity of the site lies in the fact that its effectivity implies a certain "collapse" or "irruption". If the worker is to truly appear, the regime of representation which functions on the basis of union vs. company would have to disappear. It would reveal an excess (a maximal existence) which is the life of the laborers themselves in contradistinction to their existence as labor-power. This excess of the working class is revealed negatively when we identify instances of exploitation, and positively through our imagining of communism.

Conclusion

Value can be expressed in two mutually exclusive ways, as use-value or exchange-value. A commodity is therefore split insofar as it is an object of value. It can be consumed or exchanged. Labor is a special commodity as it is the only one that produces value via its consumption. The disjunction between labor's use and exchange is the source of surplus value, and the motor of capitalist expansion. Capital is value which has the sole function of producing value, a self-expanding form of value. These are the terms in which the logic of value can be understood. In order for capital to exist, there must be the disjunction Marx identified at the heart of the value-form. This disjunction is necessarily invisible in the capitalist world, and it is precisely the excess of the worker that is its ontological support.

Taken as appearance, the commodity is a multiple and its indexing in two separate worlds. This index is either use or exchange value. Here, the formal treatment that Badiou gives to appearance shows its strength, since what is unfathomable in the experience of a single subject (that value is both use and exchange), can simply be modeled as two separate objects (M, Φ_u) and (M, Φ_e) where Φ_u is indexing by use-value and Φ_e the indexing by exchange-value. In other words, the disjunction of value can be translated as two different localizations of the same multiple. This becomes especially interesting in the case of labor, where the difference between the two indices is potentially re-introduced as surplus-value, which is paradoxically the way value reproduces itself. But how is it possible to count as appearance the very difference between two modes of appearing?

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38   Badiou 1986 p. 175

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In other words, how do we formally inscribe both the difference and unification of Φ_u and Φ_e? Recall that the indexing function can be done via different phenomenal components of a "larger" phenomena. In other words, we can define two components C_u and C_e of the phenomena of value C.

The materialist wager of Marx is to say that these two components are actually atomic. Ontological elements c identify atoms of appearance A_c, and the latter themselves map elements of our multiple (a_1, a_2, ... a_n) to their difference with c. Imagine that a single multiple is indexed twice, such that there exists two atoms of appearance which vary widely between each other. That is to say, the first atom of appearance may inscribe low intensities between the fixed element and the other elements of our multiple, while the second atom inscribes a nearly maximal (set of) intensities to the same multiple. In other words, we have a localized element of a multiple which is not very different from others in a given world, and the same element which, when localized in another way, is maximally different.

Such is the case when labor is bought on the market and then consumed for a higher output of value. On the market, it may have been bought at a low price - e.g. the labor of factory workers in China - thus attesting to its minimal degree of appearance (within the context of global capitalism), but when the product of labor is sold - e.g. as a smartphone - it "stands out" from the crowd of other commodities, its degree of appearance is maximal.

As we know, it is possible to disavow the existence of a link between exchange and use value of labor. It is the most pertinent fact of the commodity that the productive process does not leave an imprint except in superficial terms (quality, etc.). But the basis of the Marxist position is to hold fast this vanishing link. So a question we should ask regarding any phenomenology which is compatible with Marx's theory: can it count the very dis-appearance of something as an appearance? This again is handled nicely by Badiou's system, since there always exists for any world a minimum degree of appearing $()$, which is the stand-in for what is essentially invisible.

Now, imagine that there exists a third indexing function Φ_3 which corresponds to a third atom, except that it has a special relation to the previous two. Namely, this third atom is compatible with the other two atoms, combining them via a transitive relation. Thus we have two different pairs of compatible objects $((M, \Phi_1), (M, \Phi_3))$ and $((M, \Phi_2), (M, \Phi_3))$ which comprise a new composite object via the transitivity of real synthesis. The construction of this third indexing function was performed

by Marx when he pointed out that it is indeed the same multiple, labor, which serves to support two distinct appearances, use and exchange value. It is from this standpoint that we obtain a new visibility on our original multiple, the being of value.

Finally, we must consider how the properties of the transcendental (ordering, minimum, conjunction, envelope) enable the expression of intuitionistic logic[39]. Intuitionistic logic was developed as part of a general rethinking of mathematics as such. It is based on a decision that mathematical objects are ultimately productions of thought and therefore must be *constructible*. In order to satisfy the criteria of constructibility, the double negation of a statement is not necessarily the affirmation. Unlike in classical logic, proofs by contradiction are not possible, since disproving the negative of a statement is not the same as proving the statement itself. Even without this resource, intuitionistic logic is still capable of generating much of the same proofs as classical logic. For Badiou, there are classical and intuitionistic worlds, depending on the structure of the transcendental. It is possible to translate statements made in one world to the other, such that one could divide the Marxist edifice into its intuitionist and classical variant. The law of value can be formulated in the former: the exchange value of a commodity is not not-related to the labor time of its production. Marx establishes the negation of the negation of the law of value, which in a non-classical world is not the same as demonstrating the affirmative. Yet this non-relation already determines something new: the visibility of the laborer.

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39   The transcendental expresses what is known as a *Heyting algebra*. In it, the negation of a degree p is equal to the envelope of all degrees unrelated to p. The conjunction of a degree and its negative is the minimum degree. Finally, the negation of negation of a degree p is the envelope of all degrees unrelated to the negation of p, which is to say that the double negation of a degree is not equivalent, but greater than, the original. For more, see Badiou 2006 pp. 166-172

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